Production of an electron-positron plasma in a pulsar magnetosphere

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A study is made of the production of electron-positron plasma in the vacuum state ("breakdown" of the vacuum) in the presence of an inhomogeneous electric field and a strong curvilinear magnetic field. Such conditions are encountered in the magnetosphere of a rotating neutron star. A general system of kinetic equations is derived for the electrons, positrons, and γ photons in the curvilinear magnetic field with allowance for the production of electron-positron pairs and the emission of curvature and synchrotron photons. The conditions of occurrence of "breakdown" are determined, and the threshold value of the jump in the value of the electric field at the surface of the star is found. The process of multiplication of particles in the magnetosphere is investigated, and the distribution functions of the electrons, positrons, and photons are found. The extinction limit of pulsars is determined. It is shown that the theory is in agreement with observational data.

In Ref. 1, Beskin and the present authors investigated the electrodynamics of a pulsar magnetosphere filled with an electron-positron plasma of fairly high density. It was shown that a region is formed in the magnetosphere in which the magnetic field lines do not close. Along them flow electric currents $j_\parallel$ that have a major influence on the processes in the magnetosphere, determining the spindown of the neutron star and the pulsar activity. The longitudinal currents $j_\parallel$ arise only if near the surface of the star there is a strong longitudinal electric field producing a potential difference

$$\varphi=\varphi_m[1-(1-j_\parallel^2/j_c^2)^{1/2}],\quad j_c=\Omega B\cos\chi/2\pi,\qquad \varphi_m=B\Omega^2R^3\cos\chi/2c^2 \tag{1}$$

between the surface of the star and the magnetosphere. Here, R is the radius of the neutron star, B is the magnetic field at its surface, Ω is the star's rotational angular frequency, and χ is the angle between the magnetic axis of the star and Ω. Under conditions typical of pulsars, $R\sim10^6$ cm, $B\sim10^{12}$ G, $\Omega\sim10\ \text{sec}^{-1}$, and $j_\parallel\sim10^2\ \text{A/cm}^2$, the jump in the potential is very large, $\varphi\sim10^{13}$ V, and the electric field, which has a component along the magnetic field, reaches $E\sim10^9$ V/cm.

The plasma is carried outside the magnetosphere along the open field lines. It must therefore be continuously produced near the surface of the star. Production of electron-positron plasma through direct "breakdown" of the vacuum by the existing electric field is impossible—it would require too strong fields, $E\gtrsim10^{16}$ V/cm. However, if in the vacuum there is also a sufficiently strong curvilinear magnetic field the situation is radically altered and "breakdown" can occur in a much weaker electric field. Such a mechanism of electron-positron plasma generation was suggested by Sturrock[2] and developed by Ruderman and Sutherland.[3] Further details were elaborated by Tademaru[4] and others.[5] The essence of the mechanism is the acquisition by the electrons and positrons of a high energy in the electric field (1) at the surface of the star. Moving along the field lines of the curvilinear magnetic field, they radiate so-called curvature photons, whose energy is sufficient to create electron-positron pairs in the magnetic field. Newly created particles of opposite sign, trapped in the electric field (1) and moving in the opposite direction to the radiating particles, also acquire a high energy and can themselves, radiating curvature photons, produce pairs. There is a chain reaction which multiplies the electrons, positrons, and γ photons. The multiplication coefficient is increased by the fact that particles produced in a high Landau level in the magnetic field radiate synchrotron radiation—"synchrophotons," which can also produce pairs.

The main features of the kinetics of this vacuum "breakdown" mechanism are determined by the fact that it takes place in the curvilinear magnetic field. It is essential to note that only in such a field are curvature photons emitted. The curvilinearity of the field is no less important for the generation of the electron-positron pairs; for pair production is possible only if a photon intersects the magnetic field lines. But a curvature photon is emitted along the motion of the particle, i.e., along the magnetic field line, and it is only the curving of the magnetic field which makes it begin to intersect the field and gradually reach the critical angle for production, which depends on the photon energy and the strength of the magnetic field.[1)]

The aim of this paper is to construct a consistent kinetic theory of the generation of electron-positron plasma in a pulsar magnetosphere. In Sec. 1, we derive a general system of kinetic equations for the electrons, positrons, and photons in the curvilinear magnetic field and the inhomogeneous electric field. Allowance is made for production of electron-positron pairs, emission of curvature photons, and emission of synchrophotons by the produced particles. In Sec. 2, we determine the conditions of breakdown and find the threshold value of the jump φ_0 in the electric field at the surface of the star. In Sec. 3, we investigate particle multiplication in the magnetosphere, and we determine the electron and positron distribution functions and also the spectrum, directionality, and intensity of the γ radiation that is produced.

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§1. BASIC EQUATIONS

We shall describe the generation and motion of electron-positron plasma and high-energy photons in the pulsar magnetosphere by the kinetic equations

$$\frac{\partial F_\sigma^\pm}{\partial t}+\sigma c\mathbf{B}\left[\frac{\partial}{\partial \mathbf{r}}\left(\frac{(\gamma^2-1)^{1/2}}{\gamma B}F_\sigma^\pm\right)\right.$$
$$\left.\mp\frac{\nabla\psi}{B}\frac{\partial}{\partial\gamma}\left(\frac{(\gamma^2-1)^{1/2}}{\gamma}F_\sigma^\pm\right)\right]=-S+Q_N, \qquad (2)$$

$$\frac{\partial N}{\partial t}+c\frac{\mathbf{k}}{k}\frac{\partial N}{\partial \mathbf{r}}=Q_F+Q_S-D. \qquad (3)$$

Here, $F_\sigma^\pm(\gamma, \mathbf{r}, t)$ is the distribution function of the electrons and positrons with respect to the longitudinal energy $\gamma=[1+(p_\parallel/mc)^2]^{1/2}$. We have taken into account the fact that the spread with respect to the transverse momenta $p_\perp$ is unimportant, since in the strong magnetic field of the pulsar all particles are effectively in the zeroth Landau level. The factor and index $\sigma=\operatorname{sign}(\mathbf{p}_\parallel\cdot\mathbf{B})=\pm1$ characterizes the direction of the longitudinal momentum relative to the magnetic field. Further, $\psi=e\varphi/mc^2$ is the dimensionless potential of the electric field. The operator $S(F_\sigma^\pm)$ describes the scattering of the electrons and positrons when they emit photons, and Q_N the production of electron-positron pairs by high-energy photons. Finally, $N(\mathbf{k}, \mathbf{r}, t)$ is the distribution function of the photons with respect to the momenta $\mathbf{k}$, the operator Q_F describes the production of photons by fast electrons and positrons, Q_S the generation of synchrophotons, and D the annihilation of photons as a result of pair production.

We now obtain explicit expressions for the operators S, Q_N, Q_F, Q_S, D just defined in Eqs. (2) and (3). The emission of curvature photons by the electrons and positrons is due to their motion along the curvilinear magnetic field. In this case, the probability of emission of a photon with momentum k (by k it is convenient to understand the wave number nondimensionalized by division by the Compton wavelength $\lambda\!\!\!^{-}=\hbar/mc$) from a charged particle with energy γ depends on the radius of curvature ρ of the magnetic field line and is determined by the expression[7]

$$P\,dk=\frac{3^{1/2}}{2\pi}\frac{\alpha c}{\rho}\frac{\gamma}{k}\Phi\left(\frac{k}{k_c}\right)dk. \qquad (4)$$

Here, $\alpha=1/137$ is the fine structure constant, and

$$\Phi(z)=z\int_z^\infty K_{5/3}(x)\,dx, \qquad k_c=\frac{3}{2}\frac{\lambda\!\!\!^{-}}{\rho}\gamma^3, \qquad (5)$$

where $K_p(x)$ is a modified Bessel function of the second kind. We give expressions which will be helpful in what follows:

$$\Phi(z)=3\Gamma\left(\frac{5}{3}\right)\left(\frac{z}{2}\right)^{1/3}\left[1+\frac{3}{4}\left(\frac{z}{2}\right)^2+\ldots\right]$$
$$-\frac{\pi}{\sqrt{3}}z, \quad z\ll1;$$

$$\Phi(z)=\left(\frac{\pi}{2}\right)^{1/2}z^{1/2}e^{-z}\left(1+\frac{55}{72}\frac{1}{z}+\ldots\right), \quad z\gg1. \qquad (6)$$

The complete form of this function is given in Ref. 8. Using the probability density $P(k,\gamma)$ (4), we can easily obtain an expression for the total number of photons produced per unit time by the fast electrons and positrons:

$$Q_F=\int_1^\infty d\gamma[F_\sigma^+(\gamma)+F_\sigma^-(\gamma)]\int_0^\infty P(k,\gamma)\,\delta\left(\mathbf{k}-\frac{\mathbf{B}}{B}\sigma k\right)dk. \qquad (7)$$

We have here used the fact that the curvature photon moves in the same direction as the particle which produces it.

The emission of the curvature photons leads to scattering of the particles. The motion being one-dimensional, the scattering operator has the general form

$$S=\int_0^\infty\{F(\gamma)P(\gamma,k)-F(\gamma+k)P(\gamma+k,k)\}\,dk. \qquad (8)$$

Assuming that the energy of the radiated photons is small compared with the energy of the particles, $k\ll\gamma$, we can expand (8) in powers of k. Restricting ourselves, as usual, to the leading terms of the expansion, we obtain

$$S=-\frac{2}{3}\alpha\frac{c\lambda\!\!\!^{-}}{\rho^2}\frac{\partial}{\partial\gamma}\left[\gamma^4F+\frac{55}{32\sqrt{3}}\frac{\lambda\!\!\!^{-}}{\rho}\frac{\partial}{\partial\gamma}(\gamma^7F)\right]. \qquad (9)$$

It can be seen that the expansion (9) is actually in powers of $\lambda\!\!\!^{-}\gamma^2/\rho$. In a pulsar magnetosphere, the condition $\lambda\!\!\!^{-}\gamma^2/\rho\ll1$ is always well satisfied.

The probability of production of an electron-positron pair in unit time by a photon with momentum $\mathbf{k}$ moving in the magnetic field $\mathbf{B}$ is given by[9]

$$W(\mathbf{k})=\frac{3^{3/2}}{2^{9/2}}\frac{\alpha c}{\lambda\!\!\!^{-}}B_0|\sin\beta|$$
$$\times\exp\left\{-\frac{8}{3kB_0|\sin\beta|}\right\}\Theta[k|\sin\beta|-k_{\perp0}]. \qquad (10)$$

Here, β is the angle between $\mathbf{k}$ and $\mathbf{B}$, and $\Theta(x)$ is the Heaviside step function,

$$B_0=B/B_c, \quad B_c=m^2c^3/e\hbar=4{,}4\cdot10^{13}\ \mathrm{G}.$$

The expression (10) is valid when

$$kB_0|\sin\beta|\ll1, \quad \text{i.e.,} \quad k\ll(\alpha\rho/\lambda\!\!\!^{-}B_0)^{1/2}, \quad [\text{see (35)}], \qquad (11)$$

when the probability of production is exponentially small. Further, $k_{\perp0}$ in (10) is the minimum value of the component of the photon momentum orthogonal to $\mathbf{B}$ at which pair production is still possible. It can be readily determined from the laws of conservation of energy and of the longitudinal component of the momentum in production in not too strong magnetic fields ($B_0\lesssim0.1$), when the electron and positron are produced with almost equal transverse momenta.[10] Then the energy γ_0 of the particles produced and the angle θ between the direction of their momentum and the direction of the magnetic field are

$$\gamma_0=k/2, \quad \sin^2\theta=(k^2\sin^2\beta-4)/(k^2-4). \qquad (12)$$

It can be seen that production is possible only if $k^2\sin^2\beta\geqslant4$, and this determines $k_{\perp0}$ in (10):

$$k_{\perp0}=(k|\sin\beta|)_0=2. \qquad (13)$$

When $k_\perp-2\gg B_0$, the electron and positron produced are in high Landau levels. Emitting synchrotron radiation, they undergo a transition to a lower level. Under pulsar magnetosphere conditions, the de-excitation time τ is very short,

$$\tau\approx\gamma\lambdabar/\alpha c B_0^2\approx 2\cdot 10^{-19}\gamma/B_0^2 \text{ sec},$$

so that the synchrophoton production process takes place almost simultaneously with the production of the pair and at the same point of space. Moreover, in the process of emitting synchrophotons the particle does not change its longitudinal velocity, i.e.,

$$\cos^2\theta(\gamma_0^2-1)/\gamma_0^2=\text{const}=(\gamma^2-1)/\gamma^2, \tag{14}$$

where γ is the longitudinal energy of the particle after de-excitation. It follows from (14) and (12) that

$$\gamma=1/|\sin\beta|. \tag{15}$$

Thus, the number of electron-positron pairs produced per unit time by the photons is given by

$$Q_N=\int W(\mathbf{k})N(\mathbf{k})\delta\left(\gamma-\frac{1}{|\sin\beta|}\right)d\mathbf{k}, \tag{16}$$

where the production probability $W(\mathbf{k})$ is determined in accordance with (10) and (13).

We now determine the number of synchrophotons generated. We use the fact that under the conditions $B_0\lesssim 0.1$ the radiation process can be treated classically.[10] Therefore, the power radiated in the interval of wavelengths dk is (see Ref. 8, §74)

$$dI=\frac{\sqrt{3}}{2\pi}\alpha\frac{mc^3}{\lambdabar}B_0|\sin\beta|\Phi\left(\frac{k}{k_B}\right)dk,\quad k_B=\frac{3}{2}B_0\gamma_0^2|\sin\beta|, \tag{17}$$

where the function $\Phi(z)$ is determined in accordance with (5) and (6). The distribution with respect to k of the emitted synchrophotons is therefore determined by

$$G(k)=-\frac{1}{2\pi|\sin\beta|k^3mc^2}\frac{dI\,dt}{dk\,d\beta}=-\frac{\alpha\sqrt{3}}{(2\pi)^2}\frac{cB_0}{\lambdabar k^3}\Phi\left(\frac{k}{k_B}\right)\frac{dt}{d\beta}. \tag{18}$$

The derivative $dt/d\beta$ can be found from the dynamics of the change in the energy γ_0 and the angle θ of the particle in the radiation process; for it follows from (17) and (14) that

$$\frac{d\gamma_0}{dt}=-\frac{2}{3}\alpha\frac{c}{\lambdabar}\sin^2\theta(\gamma_0^2-1)B_0^2,$$

$$\frac{d\theta}{dt}=\cot\theta\frac{1}{\gamma_0(\gamma_0^2-1)}\frac{d\gamma_0}{dt}. \tag{19}$$

Since the radiation is concentrated near the generators of the "velocity cone" of the relativistic particle, $\beta=\theta$.

Substituting in (18) $(d\theta/dt)^{-1}$ from (19), we obtain

$$G(k)=\frac{3^{3/2}}{8\pi^2}\left\{k^3B_0|\sin\beta|\left[\frac{1}{\gamma^2}-\sin^2\beta\right]^{1/2}\right\}^{-1}\Phi\left(\frac{k}{k_{B_0}}\right),$$

$$k_{B_0}=\frac{3}{2}B_0|\sin\beta|\cos^2\beta\left(\frac{1}{\gamma^2}-\sin^2\beta\right)^{-1}.$$

We have here used the fact that the distribution of the emitted synchrophotons depends, by virtue of Eqs. (12) and (15), on the wave vector $\mathbf{k}'$ of the photon that produced the pair:

$$\frac{1}{\gamma}=|\sin\beta'|,$$

$$k_{B_0}=\frac{3}{2}B_0\frac{|\sin\beta|\cos^2\beta}{\sin^2\beta'-\sin^2\beta},\quad \sin^2\beta'-\sin^2\beta\geqslant\frac{4\cos^2\beta}{k'^2}.$$

Finally, the number of synchrophotons produced per unit time is determined by

$$Q_S=2\int W(\mathbf{k}')N(\mathbf{k}')G(k)\Theta\left[\sin^2\beta'-\sin^2\beta-\frac{4\cos^2\beta}{k'^2}\right]dk'$$

$$=\frac{3^{3/2}}{(2\pi)^2}\frac{1}{k^3B_0|\sin\beta|}\int\frac{W(\mathbf{k}')N(\mathbf{k}')\Phi(k/k_B)}{(\sin^2\beta'-\sin^2\beta)^{1/2}}\Theta$$

$$\times\left[\sin^2\beta'-\sin^2\beta-\frac{4\cos^2\beta}{k'^2}\right]dk'. \tag{20}$$

The quantity D, which describes the annihilation of the photons when they produce electron-positron pairs, is evidently equal to

$$D=W(\mathbf{k})N(\mathbf{k}). \tag{21}$$

Thus, the system of kinetic equations (2) and (3) is closed by the expressions (7), (9), (16), (20), and (21), which describe the scattering and production of the electrons, positrons, and photons. The equations are valid under the conditions

$$B_0\leqslant 0.1,\quad \lambdabar\gamma^2/\rho\ll 1,\quad k^2\ll\alpha\rho/\lambdabar B_0, \tag{22}$$

which in a pulsar magnetosphere, where

$$\rho\approx 7\cdot 10^7P^{1/2}\text{ cm},\quad P\leqslant 1\text{ sec},\ \gamma\sim 10^7,\quad k\sim 10^4, \tag{23}$$

are always well satisfied.

The (hydrodynamic) motion of the electrons and positrons across the magnetic field lines can be taken into account in the same way as in Ref. 1. However, for our problem it is only distances $r-R$ not too far from the pulsar surface,

$$r-R\sim R\ll c/\Omega, \tag{24}$$

for which the transverse drift of the particles is slight,[2)] that are important.

The change in the magnetic field due to the currents flowing in the pulsar magnetosphere can be ignored[1] under the conditions (24). Therefore only the Poisson equation for the longitudinal electric field in the rotating plasma is important; it has the form

$$\Delta\psi=-\frac{4\pi e^2}{mc^2}\left[\sum_\sigma\int_1^\infty(F_\sigma^+-F_\sigma^-)d\gamma-n_c\right],$$

$$\psi=e\varphi/mc^2,\quad n_c=B\Omega\cos\chi/2\pi ec=B_0\Omega\cos\chi/2\pi\alpha c\lambdabar^2. \tag{25}$$

Here n_c is the corotation charge density,[3,1] and χ is the angle between the rotation axis and the axis of the star's magnetic field, which is all that follows will be assumed to be a dipole field.

This process of plasma production takes place in a region that begins at the surface of the star and ends at comparatively large heights $z\lesssim R$. Accordingly, the boundary conditions for the kinetic equations (2) and (3) and the Poisson equation (25) must be specified on the surface $z=0$ and at $z \gtrsim R$. At $z=0$,

$$F_1^{\pm}(\gamma, t, r_\perp, z=0)=K^{\pm}(F_{-1}^{+}, F_{-1}^{-}, N^{(-)})|_{z=0};$$
$$N^{(+)}(k, t, r_\perp, z=0)=K_N(F_{-1}^{+}, F_{-1}^{-}, N^{(-)})|_{z=0}. \tag{26}$$

To be specific, we have here assumed that the vector **B** is directed from the surface of the star. Accordingly, the functions $F_1^{\pm}$ and $N^{(+)}$ describe the particles and photons moving away from the surface, and $F_{-1}^{\pm}$ and $N^{(-)}$ those moving toward it. The coefficients of multiplication of the particles, $K^{\pm}$, and of the photons, K_N, on the surface of the star are in the general case linear operators.

At large distances $z \gtrsim R$ the plasma escapes from the star, i.e.,

$$F_{-1}^{+}=F_{-1}^{-}=N^{(-)}\to 0, \quad z \gg R. \tag{27}$$

At the same time, the potential of the electric field tends to a constant value ψ_c determined by the condition of quasineutrality:

$$\int_1^\infty (F_1^{+}-F_1^{-})\,d\gamma=n_c(1-i^2)^{1/2}, \quad i\equiv j_\parallel/j_c. \tag{28}$$

The existence of the appreciable potential difference (1) between the surface of the star and the magnetosphere leads to the natural formation of a layer near the surface in which there is a strong electric field. This layer is analogous to the ordinary Langmuir double layer at the surface of bodies in a plasma. We shall therefore call it the double layer (in pulsar literature, it is frequently called the "vacuum gap"[3]). It is in the double layer that the particles acquire the high energy needed for the emission of curvature photons capable of generating electron-positron pairs. It is therefore natural to consider separately the double layer region, which determines the conditions of occurrence of the "breakdown," and the quasineutral plasma region, where the effective particle multiplication occurs.

§2. THE DOUBLE LAYER

The double layer is a region near the surface of the star in which there is a strong electric field. The characteristic length—the thickness L of the double layer—can be readily estimated on the basis of the Poisson equation (25). Considering the direction z along the normal to the layer and bearing in mind that the plasma density in the layer is low, we find from (25) that

$$L\approx(|\psi| mc^3/eB\Omega\cos\chi)^{1/2}=(|\psi|\lambda c/\Omega B_0\cos\chi)^{1/2}. \tag{29}$$

Here, $\psi=e\varphi/mc^2$ is the dimensionless potential difference (1) between the star and the magnetosphere.

The radius of curvature ρ (22) of the magnetic field lines is always much greater than the characteristic length L:

$$\rho \gg L. \tag{30}$$

Over a distance of order L along a field line the radius of curvature hardly changes, i.e., for each field line in the double layer it can be regarded as constant. But in the direction orthogonal to the field lines, in the magnetic plane, the variations in ρ are appreciable: $\rho=(16Rc/9\Omega f)^{1/2}$ (f is a dimensionless coordinate in the magnetic plane measured from the axis of the magnetic dipole). Introducing also the vertical coordinate z, we represent the magnetic field B in the double layer in the form

$$B_z=B_0=\text{const}, \quad B_f=B_0 z/\rho, \quad B_x=0, \quad \rho=\rho(f).$$

Such a representation is valid when

$$z \ll R. \tag{31}$$

Under the conditions of the double layer, the photon distribution function $N(\mathbf{k}, \mathbf{r}, t)$ takes the form $N(k, k_f, z, f, \sin k_z, t)$. We have here separated the magnitude of the wave vector, $k\approx|k_z|$, and the transverse component, $k_f\sim\beta$, since under the conditions (12) in which we are interested $\sin\beta \ll 1$ always. It is convenient to introduce the component of the wave vector transverse with respect to the magnetic field **B**:

$$y=\frac{3}{4}B_0 k\left(\frac{z}{\rho}\operatorname{sign}k_z-\frac{k_f}{k}\right). \tag{32}$$

Equation (3) for the photon distribution function then takes the form (in passing to the new variables, we have also normalized the function N in such a way that the product $N d\mathbf{k}$ remains invariant)

$$\frac{\partial N}{\partial t}+c\operatorname{sign}k_z\frac{\partial N}{\partial z}+\frac{3}{4}cB_0\frac{k}{\rho}\frac{\partial N}{\partial y}=-W(\mathbf{k})N$$
$$+\sum_{\pm}\int_1^\infty F_\sigma P(k,\gamma)\delta(y)\delta_{\sigma,\operatorname{sign}k_z}\,d\gamma. \tag{33}$$

We have here used the fact that under the conditions (30) displacement with respect to f is unimportant, so that f occurs in Eq. (33) only as a parameter through $\rho=\rho(f)$. We have also noted that the mean free path for pair production by the synchrophotons is appreciably greater than for the bulk photons; therefore, the part played by the synchrophotons in the double layer is unimportant, and they can be ignored. Initially, we also ignore the flux of γ photons knocked out from the surface of the star, i.e., we assume that $K_N=0$ in the boundary conditions (26). Then the steady solution of Eq. (33) can be written in the form

$$N(k,y,z,\operatorname{sign}k_z)=\frac{4\rho}{3cB_0k}\sum_{\pm}\int_1^\infty d\gamma F^{\pm}\left(\gamma,\sigma,z-\frac{4}{3}\frac{\rho y}{kB_0}\sigma\right)$$
$$\times P(k,\gamma)I(y,k)c(z)\Theta(y)\delta_{\sigma,\operatorname{sign}k_z}, \tag{34}$$

where

$$I(y,k)=\begin{cases}\exp\left[-\dfrac{\alpha}{\sqrt{6}}\dfrac{\rho}{\lambda k^2B_0}\displaystyle\int_{3/2B_0}^{y}y'e^{-2/y'}\,dy'\right], & y\geq 3/2B_0\\ 1, & y<3/2B_0\end{cases}, \tag{35}$$

$$c(z)=\begin{cases}\Theta\left(z-\dfrac{4}{3}\dfrac{\rho y}{kB_0}\right), & \sigma>0\\ \Theta\left(z_0-z-\dfrac{4}{3}\dfrac{\rho y}{kB_0}\right), & \sigma<0.\end{cases} \tag{36}$$

Here, $z_0>L$ is the value of the coordinate z at which the boundary condition (27) is satisfied. Thus, we see that the photon distribution function at the point z is determined by the distribution of the positrons and electrons at the point $z' = z - (4\rho y/3kB_0)\sigma$, where the photons were emitted. In addition, from the point of emission z' to z the photon distribution function is broadened monotonically with respect to the transverse momenta y, this being described by the factor $c(z)$ defined in (36).

Substituting the function N (34) in Eq. (2), we obtain closed equations for the particle distribution functions, which under steady conditions take the form

$$c\sigma\left[\frac{\partial}{\partial z}\left(\frac{(\gamma^2-1)^{1/2}}{\gamma}F^\pm\right)\mp\nabla\psi\frac{\partial}{\partial\gamma}\left(\frac{(\gamma^2-1)^{1/2}}{\gamma}F^\pm\right)\right]=-S+Q_N; \tag{37}$$

$$Q_N=\frac{2^{1/2}}{3\pi}\frac{c\alpha^2}{\lambda\!\!\!^{-} B_0{}^2}\int\limits_{3/2B_0}^{\infty}dy\int\limits_0^\infty\frac{y^2}{k^3}I(y,k)e^{-2/y}c(z)\delta\left(k-\frac{4}{3B_0}\gamma y\right)dk$$

$$\times\sum_\pm\int\limits_1^\infty d\gamma' F^\pm\left(\gamma',\sigma,z-\frac{\rho}{\gamma}\sigma\right)\gamma'\Phi\left[\frac{2}{3}\frac{\rho}{\lambda\!\!\!^{-}}\frac{k}{\gamma'^3}\right].$$

These equations can be greatly simplified. First, the diffusion spreading $\Delta\gamma$ of the energy distribution function, described by the second term in the expression (9) for S, depends on the distance z from the surface,

$$\Delta\gamma\sim\gamma\varepsilon_1,\quad \varepsilon_1=\left(\frac{2}{3}\frac{\alpha\gamma}{\rho}\right)^{1/2}\frac{\lambda\!\!\!^{-}}{\rho}\gamma^2 z^{1/2};\quad \varepsilon_1\ll 1,$$

and at scales of the order of the thickness L (29) of the double layer it is negligible under the ordinary pulsar conditions (23). Equally unimportant is the energy spread due to the particle production Q_N:

$$\Delta\gamma\sim\gamma\varepsilon_2,\quad \varepsilon_2=\frac{3}{4}\frac{\lambda\!\!\!^{-}}{\rho}\gamma^2,\quad \varepsilon_2\ll 1.$$

Thus, in the zeroth approximation in the small parameters ε_1 and ε_2 the energy of the particles in the double layer is completely determined by the action of the electric field and the deceleration (first term in S). This means that the energy distribution functions of the particles have the form

$$F^\pm=n^\pm(z)\delta(\gamma-\gamma_1{}^\pm(z)). \tag{38}$$

Then, from the kinetic equation (37), taking into account (38) and (9), we find

$$n^+(z)=n^+(0)\gamma_1{}^+((\gamma_1{}^+)^2-1)^{-1/2},$$
$$n^-(z)=n^-(z_m)\gamma_1{}^-((\gamma_1{}^-)^2-1)^{-1/2}, \tag{39}$$

$$\frac{d\gamma_1{}^\pm}{dz}=\mp\frac{d\psi}{dz}-\frac{2}{3}\frac{\alpha\lambda\!\!\!^{-}}{\rho^2}(\gamma_1{}^\pm)^4,\quad \gamma_1{}^+(z=0)=\gamma_1{}^-(z=z_m)=1. \tag{40}$$

Note that, as will be shown below, the second term in (40) is not important under the conditions actually found in pulsars.

In (40), to be specific, we have assumed that the potential of the surface of the star with respect to the plasma is positive: $\psi_0>0$; this corresponds to $n_c>0$, i.e., $\mathbf{B}\cdot\mathbf{\Omega}<0$. In this case, the positrons are accelerated by the field in the direction of the magnetosphere, and the electrons toward the star, and this is taken into account in Eqs. (39). In the opposite case, $\mathbf{B}\cdot\mathbf{\Omega}>0$, we have

$$n^-(z)=n^-(0)\gamma_1{}^-((\gamma_1{}^-)^2-1)^{-1/2},$$
$$n^+(z)=n^+(z_m)\gamma_1{}^+((\gamma_1{}^+)^2-1)^{-1/2}. \tag{41}$$

It is important to emphasize that the particle density $n^-(z_m)$ in Eq. (39) [and accordingly $n^+(z_m)$ in (41)] is the density of the particles that are produced near the point z_m and, under the influence of the electric field, change the direction of their velocity (for these $\sigma<0$). To determine this quantity, it is necessary to know the distribution function of these particles to an accuracy $\Delta\gamma/\psi_0\approx\varepsilon_2$. In addition, the function $I(y, k=4\gamma y/3B_0)$ (35), which describes the distribution of the curvature photons with respect to the angles y, can be conveniently approximated with good accuracy by a step function:

$$I(y, k=4\gamma y/3B_0)=\Theta(\Lambda^{-1}-y),\quad \Lambda={}^1/_2\ln(3^{3/2}\alpha\rho B_0/2^{17/2}\lambda\!\!\!^{-}\gamma^2).$$

Substituting in the expression (37) for Q_N the distributions of the fast particles in the form (38) and (39), we obtain for the distribution functions of the produced particles at $z\sim z_m$

$$F^\pm(\gamma,z,\sigma=\pm1)=\frac{\sqrt{3}}{2\pi}\frac{\alpha n^+(0)}{\rho}\frac{\gamma}{(\gamma^2-1)^{1/2}}\int\limits_0^z\frac{1}{\gamma'}\gamma_1{}^+\left(z'-\frac{\rho}{\gamma'}\right)$$
$$\times\Phi\left[\frac{4\varkappa\rho}{3\lambda\!\!\!^{-}}\gamma'\left(\gamma_1{}^+\left(z'-\frac{\rho}{\gamma'}\right)\right)^{-3}\right]dz'+n^+(0)\delta(\gamma-\gamma_1{}^+(z)),$$

$$\gamma'=\gamma+\gamma_1{}^\pm(z')-\gamma_1{}^\pm(z),\quad \varkappa={}^2/_3(B_0\Lambda)^{-1}. \tag{42}$$

For particles produced at $z\sim0$ we will accordingly have

$$F^\pm(\gamma,z,\sigma=\pm1)=\frac{\sqrt{3}}{2\pi}\frac{\alpha n^-(z_m)}{\rho}\frac{\gamma}{(\gamma^2-1)^{1/2}}\int\limits_z^{z_m}\frac{1}{\gamma'}\gamma_1{}^-\left(z'+\frac{\rho}{\gamma'}\right)$$
$$\times\Phi\left[\frac{4\varkappa\rho}{3\lambda\!\!\!^{-}}\gamma'\left(\gamma_1{}^-\left(z'+\frac{\rho}{\gamma'}\right)\right)^{-3}\right]dz'+n^-(z_m)\delta(\gamma-\gamma_1{}^-(z)). \tag{43}$$

It can be seen from expressions (42) and (43) that effective production of electron-positron pairs occurs over a scale Δz small compared with the complete thickness L of the double layer:

$$\Delta z/L\sim(L/\rho)\gamma\varepsilon_2\sim10^{-1}. \tag{44}$$

This means that the process of plasma multiplication in the layer is linear in nature. For each accelerated positron K_m electrons are created and reflected backward by the electric field near the point $z=z_m$:

$$n^-(z_m)=K_m n^+(0). \tag{45}$$

Similarly, near the boundary of the star an electron accelerated by the field generates K_1 positrons. Then, using the boundary condition (26), we obtain

$$n^+(0)=K_0 n^-(z_m),\quad K_0=K_1+K^\pm; \tag{46}$$

where K^+ are the multiplication coefficients of particles due to knocking out of particles of the opposite sign from the surface of the star [see (26)]. In (46), it is assumed that the energy of the particles that are knocked out is small compared with the energy of the particles incident on the surface. From (45) and (46), we obtain the following condition for the existence of a steady process of plasma generation:

$$K_0K_m=1,\quad K_m=1/K_0. \tag{47}$$

Expressions for the coefficients K_m and K_1 can be obtained by integrating the distribution functions (42) and (43) of the secondary particles over the energy. The result is

$$K_1=\frac{\sqrt{3}}{2\pi}\frac{\alpha}{\rho}\int\limits_{z_m-t_m}^{z_m} dt\gamma_1^-(t)\left\{Y\left[\frac{4\rho^2(\gamma_1^-(t))^{-3}\varkappa}{3\lambda(z_2(z_m,t)-t)}\right]\right.$$
$$\left.-Y\left[\frac{4\rho^2(\gamma_1^-(t))^{-3}\varkappa}{3\lambda(z_1(z_m,t)-t)}\right]\right\}. \tag{48}$$

$$K_m=\frac{\sqrt{3}}{2\pi}\frac{\alpha}{\rho}\int\limits_0^{t_m} dt\gamma_1^+(t)\left\{Y\left[\frac{4\rho^2(\gamma_1^+(t))^{-3}\varkappa}{3\lambda(z_2(z_m,t)-t)}\right]\right.$$
$$\left.-Y\left[\frac{4\rho^2(\gamma_1^+(t))^{-3}\varkappa}{3\lambda(z_1(z_m,t)-t)}\right]\right\}, \tag{49}$$

where $z_{1,2}(z,t)$ are the roots of the equation

$$z_{1,2}-\rho\Big/\left(1+\gamma_1^\pm\left(\frac{z}{z_m-z}\right)-\gamma_1^\pm(z_{1,2})\right)=t,$$

and t_m is the value of t at which $z_1=z_2$. In (48) and (49),

$$Y(x)=\int\limits_x^\infty(\eta-x)K_{5/3}(\eta)\,d\eta$$

$$=\begin{cases}5\pi/3-9\Gamma(5/3)x^{1/3}/2^{1/3}+\pi x/\sqrt{3}+\dots, & x\ll1\\ (\pi/2x)^{1/2}e^{-x}, & x\gg1.\end{cases} \tag{50}$$

The values of this function are as follows:

x:	0	0,01	0.05	0,1	0,2	0,5	1,0	1,5	2,0	3,0	4,0	5,0
$Y(x)$:	5.24	3.87	2.95	2.42	1.82	0,99	0,45	0,23	0,12	0,037	0.012	0,0039

Equations (40) must be solved simultaneously with the Poisson equation (25), which, the double layer being assumed thin, takes the form

$$\frac{d^2\psi}{d\xi^2}=1-(n^+-n^-),\quad \xi=\frac{z}{D_c},\quad D_c=\left(\frac{mc^2}{4\pi n_ce^2}\right)^{1/2}. \tag{51}$$

The densities n^+ and n^- are expressed here in units of n_c (25). In accordance with (26) and (28), the boundary conditions for Eq. (51) are

$$\psi(0)=\psi_0,\quad \psi(\xi_m)=0,\quad d\psi/d\xi|_{\xi=\xi_m}=0,\quad \xi_m=z_m/D_c. \tag{52}$$

The last condition follows from the necessity of smooth matching of the potential in the double layer, $\xi<\xi_m$, and in the quasineutral region $\xi>\xi_m$ (28). Bearing in mind that by virtue of (44) the densities n^+ and n^- in the layer are constant, we can readily find the solution of Eq. (51) with the boundary conditions (52):

$$\psi=(1-n^++n^-)(\xi-\xi_m)^2/2,\quad \xi<\xi_m.$$

Here

$$\xi_m=(2\psi_0)^{1/2}[1-(n^+-n^-)]^{-1/2},$$

and ψ_0 is the potential difference between the star and the magnetosphere. The electron, n^-, and positron, n^+, concentrations can be conveniently expressed in terms of the current density $i=j_\parallel/j_c$:

$$n^+=i/(1+K_m),\quad n^-=iK_m/(1+K_m).$$

For the thickness of the double layer, we then obtain

$$z_m=D_c(2\psi_0)^{1/2}(1-pi)^{-1/2},\quad p=(1-K_m)/(1+K_m).$$

Substituting this expression in (48) and (49), we obtain from (47) with allowance for (46) an equation that determines the threshold value ψ_0 of the field potential.

The results of its solution are conveniently represented in the form

$$\psi_0=\frac{2^{5/7}}{3^{4/7}}\frac{\rho^{4/7}(B_0\Lambda)^{-2/7}}{\lambda^{2/7}D_c^{2/7}}(1-pi)^{1/7}b^{-2/7}, \tag{53}$$

where b is a function of the dimensionless parameter d:

$$d=\frac{2^{4/7}\alpha}{\pi\cdot3^{5/14}}\rho^{-1/7}\lambda^{-3/7}D_c^{4/7}(1-pi)^{-2/7}(B_0\Lambda)^{-3/7}. \tag{54}$$

The dependence of b on d for different values of $K^\pm$ is shown in Fig. 1. It can be seen that in the range $10\lesssim d\lesssim30$ of actual interest for pulsars $b(d,K^\pm)$ varies little and does not differ strongly from its value when $d\gg1$. The dependence of b on $K^\pm$ for $d\gg1$ is shown in the same figure. The numerical expressions for ψ_0 and z_m have the form

$$\psi_0=1.1\cdot10^7\rho_7^{4/7}(PB_{12}\cos\chi)^{-1/7}(1-pi)^{1/7}(\Lambda/8)^{-2/7}b^{-2/7};$$
$$z_m=9.5\cdot10^3\rho_7^{2/7}B_{12}^{-4/7}P^{3/7}(\cos\chi)^{-3/7}(1-pi)^{-3/7}(\Lambda/8)^{-1/7}b^{-1/7}. \tag{55}$$

Here $\psi_0=e\varphi_0/mc^2$ is expressed in dimensional units, z_m in cm, ρ_7 in 10^7 cm, B_{12} in 10^{12} G, and

$$\Lambda=7.66-\frac{3}{14}\ln\rho_7-\frac{1}{14}\ln B_{12}$$
$$+\frac{3}{7}\ln\left(\frac{P}{(1-pi)\cos\chi}\right)-\frac{1}{7}\ln b.$$

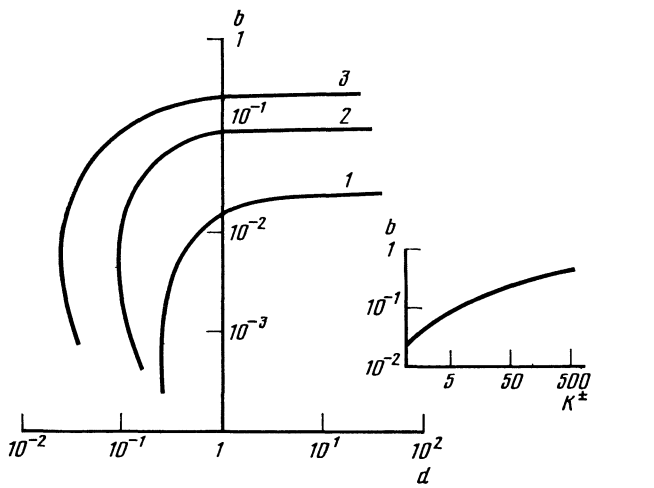

FIG. 1. Graph of b (53) as a function of the parameter d (54): 1) $K^\pm=0$, 2) $K^\pm=5$, 3) $K^\pm=50$. The insert shows b as a function of $K^\pm$ for $d\gg1$.

It can be seen from (55) that for pulsars the characteristic "breakdown" potential, i.e., the potential needed for steady plasma generation, is $\varphi \sim 10^{13}$ V, and the thickness of the double layer is $L \sim 10^2$ m. Comparing the result (55) of the exact theory with the approximate estimate of ψ_0 obtained by Ruderman and Sutherland[3] from the condition of pair generation, we see that the numerical coefficient has been changed.[3)] In addition, the expression (55) describes the influence on the breakdown conditions of the interaction of the electrons and positrons with the pulsar surface: $b = b(K^{\pm})$. Note also that the influence of the γ photons knocked out of the surface [$K_N \neq 0$ in (26)] is in fact taken into account by the same dependence of b on $K^{\pm}$; for it is readily seen from (35) that when the photons are emitted from the surface at an arbitrary angle to the magnetic field they generate pairs near the surface ($z \lesssim 10^2$–10^3 cm), and this leads merely to a change in the effective coefficient $K^{\pm}$ ($K^{\pm}_{\rm eff} = K^{\pm} + K_N$). The only exception is in the case of photons that are emitted exactly along the magnetic field, an improbable event.

It is also very important that the potential ψ_0 depends in accordance with (55) on the longitudinal current i. This means that, strictly speaking, (55) does not establish the value of the potential ψ_0 but the connection between ψ_0 and i. Another relationship between ψ_0 and i can be found from the dynamics of the current in the magnetosphere[1] and has the form (1). Thus, the exact values of the breakdown potential ψ_0 and the current i are found by simultaneous solution of Eqs. (1) and (55). The current $i(f)$ is determined from

$$\frac{1-(1-i^2)^{1/2}}{(1-pi)^{1/7}} = 0{,}85\rho_7^{4/7}(f)P^{13/7}B_{12}^{-3/7} \times (\cos\chi)^{-8/7} b^{-2/7}\left(1-\frac{f}{f^*}\right)^{-1}\left(\frac{\Lambda}{8}\right)^{-2/7},$$

and from this value of the current the potential $\psi_0(f)$ is then determined.

We emphasize that the potential ψ_0 found in this manner may not be achieved at all parameters of the pulsar. A restriction is associated with violation of the assumption that the double layer is thin, since the open field line region, which is where the double layer exists, covers only a polar cap of the pulsar[4)] measuring

$$R_0 = R(2\pi R f^*(\chi)/Pc)^{1/2}, \quad 1{,}59 \leqslant f^*(\chi) \leqslant 1{,}95.$$

The solution obtained above is valid only if $z_m \ll R_0$ [here, $f^*(\chi)$ is a numerical parameter characterizing the polar cap[1]]. But if z_m approaches R_0 in magnitude, it is necessary to solve not Eq. (51) but the complete Poisson equation (25). The corresponding solution shows that growth of the potential ψ_0 ceases when z_m reaches the value R_0/μ_1, where $\mu_1 \approx 2.4$ is the first root of the zeroth Bessel function. The resulting restriction leads to the condition

$$B_{12}P^{-15/8} > 0{,}6. \tag{56}$$

This condition determines the possibility of steady plasma generation in the magnetospheres of pulsars, i.e., determines their extinction limit. The extinction limit (56) is shown in Fig. 2, in which we also show all pulsars, their magnetic fields B being given in accordance with the values currently adopted.[11,12]

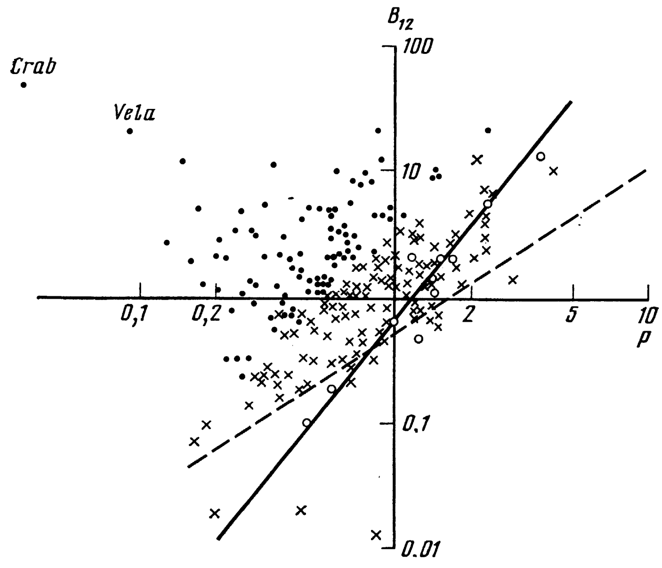

FIG. 2. Extinction boundary of pulsars in the (B_{12}, P) diagram with, to the left of it, the region of steady plasma generation. The black dots represent "young" pulsars ($Q<1$), the crosses "old" pulsars ($Q>1$), and the open circles represent pulsars having irregular emission.[12] The broken line represents the limit $\psi_0 = \psi_{0c}$ (63), to the right of which the energy released in the magnetosphere is small.

It follows from comparison of (56) with the observed boundary of the pulsar distribution that $b(K^{\pm}) \approx 0.25$, and this means that the mean coefficient of particle knockout from the surface is $K^{\pm}_{\rm eff} \approx 10^2$.

A more detailed comparison of the theory with observational data, which requires the particular structure of the current region (see Ref. 12), goes beyond the scope of the present paper.

§3. PLASMA MULTIPLICATION

We have considered above the production of electrons and positrons in the double layer at the surface, $z \lesssim z_m$, of the neutron star. In the quasineutral region $z > z_m$ intense multiplication of the electron-positron plasma occurs. The source of the multiplication is the stream of fast positrons (or electrons) accelerated in the double layer to energies $\gamma \sim 10^7$. These primary particles, moving along the field lines of the curvilinear magnetic field, radiate curvature photons, which produce the plasma.

We consider therefore first of all the distribution function $F_0(t, z, f, \gamma)$ of the primary particles. Its change due to scattering on emission of the curvature photons is described under steady conditions in accordance with (2) and (9) by the equation

$$\frac{\partial F_0}{\partial z} = \frac{2\alpha}{3}\frac{\lambda\!\!\!^{-}}{\rho^2}\frac{\partial}{\partial\gamma}\left[\gamma^4 F_0 + \frac{55}{32\sqrt{3}}\frac{\lambda\!\!\!^{-}}{\rho}\frac{\partial}{\partial\gamma}(\gamma^7 F_0)\right]. \tag{57}$$

In accordance with (38), the boundary conditions for Eq. (57) have the form

$$F_0|_{z=z_m} = n_0\delta(\gamma-\gamma_0), \quad n_0 = in_c/(1+K_m),$$

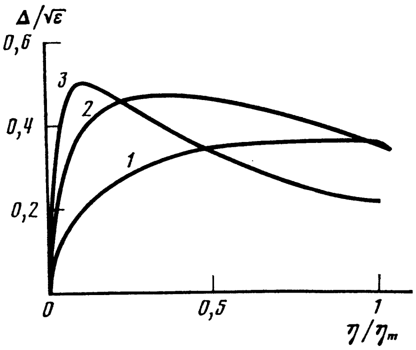

FIG. 3. Width of the distribution function (60) of the primary particles with respect to the energy as a function of the height above the pulsar surface η/η_m (58): 1) $\eta_m = 0.1$, 2) $\eta_m = 0.5$, 3) $\eta_m = 2$.

where γ_0 depends on the jump $|\psi_0|$ in the potential in accordance with (40); usually, $\gamma_0 = |\psi_0|$.

It is convenient to introduce the dimensionless variables

$$\Gamma=\frac{\gamma}{\gamma_0},\quad \eta=\frac{2}{3}\alpha\lambda\gamma_0^3\int_{z_m}^{z}\frac{dz'}{\rho^2(z')}. \tag{58}$$

We have here noted that the radius of curvature $\rho(z)$ varies with z, and for $z/R \ll (c/R\Omega)^{1/3}$

$$\rho(z)=\rho_0\left(1+\frac{z}{R}\right)^2,\quad \rho_0=\frac{4}{3}\left(\frac{cR}{\Omega f}\right)^{1/2}\quad (0<f<f^*).$$

Then

$$\frac{\partial F_0}{\partial\eta}=\frac{\partial}{\partial\Gamma}\left[\Gamma^4F_0+\varepsilon\frac{\rho_0}{\rho(\eta)}\frac{\partial}{\partial\Gamma}(\Gamma^7F_0)\right],\quad \varepsilon=\frac{55}{32\sqrt{3}}\frac{\lambda\gamma_0^2}{\rho_0}. \tag{59}$$

In accordance with (23), the parameter ε is always small under pulsar conditions, $\varepsilon\sim 10^{-4}$. This means that the diffusion broadening of the distribution function, $\sim\varepsilon^{1/2}$, is small, so that the solution of Eq. (57) can be sought in the form

$$F_0=\frac{n_0}{(\pi\Delta^2(\eta))^{1/2}}\exp\left[-\frac{(\Gamma-\bar\Gamma(\eta))^2}{\Delta^2(\eta)}\right]. \tag{60}$$

Substituting (60) in (59), expanding in powers of $\varepsilon^{1/2}$, and retaining the leading terms ε^{-1} and $\varepsilon^{-1/2}$, we obtain

$$\bar\Gamma=(1+3\eta)^{-1/3},$$

$$\Delta^2(\eta)=\frac{4\varepsilon}{(1+3\eta)^{8/3}}\int_0^{\eta}(1+3\eta')^{1/3}\left(1-\frac{\eta'}{\eta_m}\right)d\eta', \tag{61}$$

where η_m is the maximum value of η:

$$\eta_m=\frac{2}{9}\frac{\alpha\lambda R\gamma_0^3}{\rho_0^2}=\frac{1}{8}\frac{\alpha\lambda\gamma_0^3\Omega}{c}f.$$

The function $\Delta(\eta)$, which describes the broadening of the distribution function F_0, is shown in Fig. 3 for different values of the parameter η_m. It can be seen that initially the broadening increases with η, and therefore with z, in proportion to $z^{1/2}$, but then reaches a maximum Δ_m, after which it decreases. The maximal value Δ_m is reached at the point $\bar\eta$ and is

$$\Delta_m=\varepsilon^{1/2}\begin{cases}(12\eta_m/5)^{1/2}, & \bar\eta=\eta_m-(24/5)^{1/2}\eta_m^{3/2} \quad 3\eta_m\ll 1\\ 1/2, & \bar\eta=2^{3/4}-1, \quad 3\eta_m\gg 1.\end{cases}$$

It is important that at large values of z, when the parameter η reaches the limiting value η_m, the deceleration of the particles ceases, and the distribution function acquires the steady form (60) with parameters $\bar\Gamma(\eta_m)$, $\Delta(\eta_m)$ given by (61). It follows from this that the total energy expended by the primary particles on radiating the curvature photons,

$$E=n_0\gamma_0[1-(1+3\eta_m)^{-1/3}], \tag{62}$$

depends strongly on the parameter η_m. For $\eta_m \gtrsim 1$, we have $E\sim n_0\gamma_0$. But if $\eta_m \ll 1$, then $E = n_0\gamma_0\eta_m \sim \gamma_0^4$. It can be seen directly from this that γ_0, i.e., the jump $|\psi_0|$ in the potential between the surface of the star and the magnetosphere, has a critical value $|\psi_{0c}|$, determined by the condition $\eta_m \approx 1/3$:

$$|\psi_{c}|\approx 3\cdot 10^7 P^{1/3}(f^*/f)^{1/3}. \tag{63}$$

If $|\psi_0| > |\psi_{0c}|$, an appreciable energy, of the order of the complete energy of the primary beam, is expended on plasma generation. But if $|\psi_0| < |\psi_{0c}|$, the energy used on plasma generation decreases very rapidly, as $(\psi_0/\psi_{0c})^4$, and as a result the generation process must be strongly suppressed even when $\psi_0/\psi_{0c} \lesssim 0.3$. The limit $\psi_0 = \psi_{0c}$, where ψ_0 is determined in accordance with (1) and (55), is shown in Fig. 2 by the broken line ($B_{12}P^{-4/3} = 0.5$). It can be seen that for $\psi_0 < \psi_{0c}$ the number of observed pulsars is reduced. The reason for this is not the extinction of the pulsars but the decrease of their radiation associated with the sharp decrease in the energy released in the magnetosphere.

We now consider the process of photon multiplication. The flux of primary particles gives curvature radiation whose spectrum N_0 in the region of the magnetosphere not too far from the surface of the neutron star [condition (31)] is described by the expression (34) obtained in the previous section. Substituting in it the distribution function (60) and integrating over γ, we find

$$N_0=\frac{2}{\pi\sqrt{3}}\frac{n_0\alpha\gamma_0}{B_0k^2}\Phi\left(\frac{k}{k_c}\right)I(y,k)\Theta(k_z)\Theta(y)c(z). \tag{64}$$

It can be seen from this that the energy spectrum of the curvature photons is a power spectrum, $N_0\sim k^{-5/3}$, up to $k\approx 0.3k_c$ (5). For $k > k_c$, the function $N_0(k)$ decreases exponentially. The distribution of the curvature photons with respect to the angles, or rather the variable y defined in (32), is constant for $y < y^*(k)$ and decreases rapidly for $y > y^*$ with width $\Delta y \approx y^{*2}$, where

$$y^*(k)=\left(\Lambda-\frac{3}{2}\ln\Lambda\right)^{-1},$$

$$\Lambda(k)=\frac{1}{2}\ln\left(\frac{\alpha\rho}{2\sqrt{6}\lambda B_0k^2}\right),\quad \Lambda\gg 1.$$

We have here used the fact that under the conditions (22) of interest here, $B_0 < 2\Lambda^{-1}/3$.

The curvature photons generate electron-positron pairs and simultaneously synchrophotons. The photon mean free path l with respect to pair production can be determined from the condition $l \approx 4\rho y^*/3kB_0$ (32) or

$$l=4\rho/3kB_0\Lambda(k). \qquad (65)$$

Synchrophoton multiplication is a cascade process. The curvature photons generate pairs and synchrophotons of the first generation (N_1). Their energy is $\Lambda(k)$ times lower than the energy of the curvature photons, and accordingly the mean free path l given by (65) is Λ times greater. The first-generation synchrophotons also generate pairs and simultaneously with them synchrophotons of the second generation N_2, etc. Thus, the total distribution function N of the photons can be represented in the form of a sum:

$$N=N_0+N_1+N_2+\dots. \qquad (66)$$

Substituting the expansion (66) in Eqs. (3), (7), (20), and (21) and taking into account (33) and (34), we arrive at the following closed chain of time-independent equations for the synchrophotons:

$$\frac{\partial N_i}{\partial z}+\frac{3}{4}\frac{kB_0}{\rho y}\frac{\partial}{\partial y}[(y^2-\xi^2)^{1/2}N_i]$$

$$=-\frac{3^{1/2}}{2^{5/2}}\frac{\alpha}{\lambda k}ye^{-2/y}N_i\Theta\left(y-\frac{3}{2}B_0\right)+Q_S(N_{i-1}); \qquad (67)$$

$$Q_S=\frac{3^2}{\pi^2 2^{9/2}}\frac{\alpha}{\lambda B_0}\frac{1}{k^2y(y^2-\xi^2)^{1/2}}\int y'^2e^{-2/y'}\left[y'^2-\frac{k'^2}{k^2}y^2\right]^{-1/2}$$
$$\times\Phi\left\{\frac{8}{9B_0^2}\frac{k^2}{k'^2y}\left[y'^2-\frac{k'^2}{k^2}y^2\right]\right\}\Theta\left[y'^2-\frac{k'^2}{k^2}y^2-\frac{9}{4}B_0^2\right]$$
$$\times N(k',y',\xi')\,dk'\,dy'\,d\xi',\quad i=1,2,3\dots. \qquad (68)$$

Here, as in (33), we have gone over to the angular variable y defined in (32) (we have assumed that sign $k_z>0$). In addition, since the synchrophotons are produced with all angles φ with respect to the direction of the magnetic field with equal probability, the distribution function now also depends on the angle φ. This dependence is taken into account in (67) by means of the quantity $\xi=y\sin\varphi$, which is conserved in the photon motion and therefore occurs in Eq. (67) as a parameter.

The first term on the right-hand side of Eq. (67) describes the annihilation of the photons due to pair production; the second is the source of the synchrophotons and is determined by the photons of the previous generation. The curvature photons N_0 (64) are the source of the first generation of synchrophotons. It is not difficult to write down the general solution of the linear equation (67). In doing so, we must bear in mind that, as can be seen from (64), absorption of photons occurs only for $y>3B_0/2$; moreover, they are produced with the same angles β as the photons of the previous generation that produce them, i.e., $y\sim\Lambda^{-2}$. Then for fields not too weak, $B_0>2\Lambda^{-2}/3$, the solution of Eq. (67) for $y>3B_0/2$ has the form

$$N_i=N_i^0\left(z-\frac{4}{3}\frac{\rho}{kB_0}(y^2-\xi^2)^{1/2},k\right)(y^2-\xi^2)^{-1/2}$$
$$\times\exp\left\{-\frac{\alpha\rho}{6^{1/2}\lambda k^2B_0}\int_{3/2B_0}^{y}\frac{y'^2e^{-2/y'}}{(y'^2-\xi^2)^{1/2}}dy'\right\}\Theta\left(\frac{3}{2}B_0-|\xi|\right). \qquad (69)$$

Substituting (69) in the expression (68) for Q_S and integrating over y' and ξ' and remembering that the exponential contains a rapidly varying function, we obtain

$$Q_S=\frac{3^{5/2}}{4\pi^2\rho}\frac{1}{k^2y(y^2-\xi^2)^{1/2}}\int_0^\infty k'^2\Phi\left(\frac{2k^2}{k'^2y}\right)\exp\left\{-\frac{\alpha\rho}{6^{1/2}\lambda k'^2B_0}\right.$$
$$\left.\times\int_{y_1}^{y_2}y'e^{-2/y'}dy'\right\}N^0\left(z-\frac{4}{3}\frac{\rho}{kB_0}y,k'\right)dk',$$
$$y_1=\frac{3}{2}B_0,\quad y_2=\left(\frac{k'^2}{k^2}y^2+\frac{9}{4}B_0^2\right)^{1/2}.$$

Further, integrating Eq. (67) over y for constant first argument N_i^0 in (69), we find the distribution function of the synchrophotons for $y<3B_0/2$:

$$N_i=\frac{4\rho}{3kB_0}(y^2-\xi^2)^{-1/2}\int_\xi^y y'Q_S\left(y',z'+\frac{4}{3}\frac{\rho}{kB_0}(y'^2-\xi^2)^{1/2}\right)dy'. \qquad (70)$$

Matching of the functions (69) and (70) at $y=3B_0/2$ determines the function N_i^0 in (69):

$$N_i^0=\frac{3^{3/2}}{\pi^2B_0}\frac{1}{k^3}\int_{4k/3B_0a}^{\infty}dk'k'^2N_{i-1}(k')Y\left(\frac{ka}{k'}\right), \qquad (71)$$

where the expression $Y(x)$ was determined in the previous section by Eq. (50),

$$a=\frac{4}{3B_0}\left[\left(\frac{2}{3\Lambda B_0}\right)^2-1\right]^{-1/2}\approx\Lambda\gg1.$$

Since $Y(x)$ decreases exponentially when $x>1$, the solution of (71) can be represented to lowest order in the parameter $a^{-1}\sim\Lambda^{-1}$ in the form of "steps":

$$N_i^0=\frac{\Gamma(5/3)}{\pi3^{5/6}}\frac{\alpha n_0}{B_0^2}\left(\frac{\rho}{\lambda}\right)^{1/3}k_c^{4/3}\left(\frac{3^{3/2}}{\pi B_0}\right)^i\frac{1}{k^3}M_i^0\left(\frac{ka^i}{k_c}\right)$$
$$\times[\Theta(k-k_ca^{-i-1})$$
$$-\Theta(k-k_ca^{-i})],\quad k_c=\frac{3}{2}\frac{\lambda}{\rho}\gamma_0^3. \qquad (72)$$

Here, the functions $M_i^0(x)$ with different i are related by the integral equation

$$M_i^0(x)=\int_x^1\frac{dy}{y}M_{i-1}^0(y)Y\left(\frac{x}{y}\right),\quad x>a^{-1},$$

where M_0^0 corresponds to the distribution function of the curvature photons: $M_0^0=x^{4/3}$. The functions $M_i^0(x)/x^3$ are shown in Fig. 4. It can be seen that with increasing i the functions $M_i^0(x)$ become ever steeper and are concentrated near small x. This means that the spectrum of the synchrophotons is becoming softer.

As regards the dependence of $N(k,y,z)$ on the angle y and the coordinate z for $y<3B_0/2$, it is given by an expression that follows from (70);

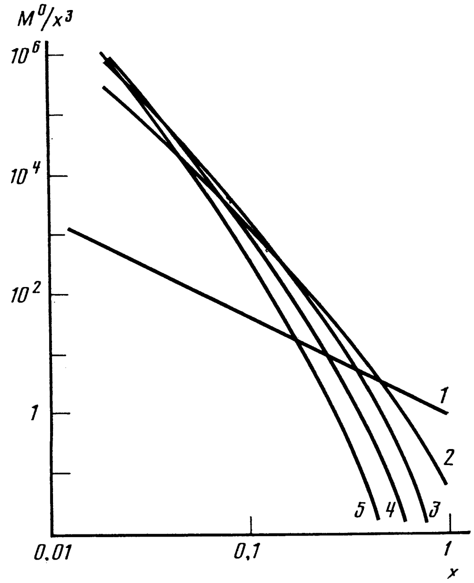

FIG. 4. The distribution functions of the curvature photons (64) and synchrophotons of different generations (72). The straight line 1 corresponds to the distribution of the curvature photons, $\sim K^{-5/3}$; 2) first generation synchrophotons $\approx K^{-3.5}$; 3) second generation $\approx K^{-4.2}$; 4) third generation $\approx K^{-5.0}$; 5) fourth generation $\approx K^{-5.7}$.

$$N(k, y, z, \xi)=\frac{\Theta(z')}{(y^2-\xi^2)^{1/2}}$$
$$\times\begin{cases} N_i^0(k), & \\ \int\limits_{k(2/y)^{1/2}}^{2k/ay} dk'k'^2 N_0(k')Y\left(\frac{2k^2}{k'^2 y}\right)+\int\limits_{2k/ay}^{\infty} dk'k'^2 N^0(k')Y\left(\frac{ak}{k'}\right); & y>2/a^2, \\ & y<2/a^2. \end{cases} \tag{73}$$

$$z'=z-\frac{4}{3}\frac{\rho}{kB_0}(y^2-\xi^2)^{1/2}.$$

Thus, N varies smoothly in the region $0<y<2/a^2$, remains constant for $2/a^2<y<y^*$, and then turns sharply at $y>y^*$, since here strong absorption of photons due to pair production commences.

It must be emphasized that the treatment has been given here for small distances from the surface of the star: $z\ll R$. With increasing distance $z\gtrsim R$, the magnetic field B_0 and the radius of curvature ρ begin to change appreciably:

$$B_0=B_0^0(1+z/R)^{-3},\quad \rho=\rho_0(1+z/R)^2.$$

The decrease in the field and the increase in the radius of curvature lead to a cutoff of the photon spectrum at $k=k_{\min}<k_c$. To find $k_{\min}$, we determine the energy contained in the secondary plasma and the photons, and we equate it to the total energy (62) lost by the primary beam. We determine first the spectrum of the secondary electrons and positrons; it follows from (2) that

$$\frac{\partial F^\pm}{\partial z}=\frac{3^{1/2}}{2^{5/2}}\frac{\alpha}{\lambdabar}\int\frac{y^2e^{-2/y}}{k}N(k,y,\xi)\Theta\left(y-\frac{3}{2}B_0\right)$$
$$\times\ \delta\left(\gamma-\frac{3}{4}\frac{kB_0}{y}\right)dk\,dy\,d\xi.$$

Substituting N from (69) and integrating, we obtain

$$\frac{\partial F^\pm}{\partial z}=\frac{4}{\rho\Lambda^2}\gamma N^0\left(k=\frac{4\gamma}{3B_0\Lambda}, z-\frac{\rho}{\gamma}\right)\Theta\left(\gamma-\frac{\rho}{z}\right). \tag{74}$$

It can be seen that the spectrum of the particles is similar to the spectrum of the photons, the only difference being in their exponents, which differ by unity—the particle spectrum is harder. The first generation of particles

$$\gamma_{max}a^{-1}<\gamma<\gamma_{max}=\frac{9}{8}\frac{\lambdabar}{\rho}\gamma_0^3B_0\Lambda(k_c)$$

has power spectrum $\gamma^{-2/3}$. It is easy to find the densities $n_1^\pm$ of these particles and their energy densities $E_1^\pm$:

$$n_1^\pm=\frac{3^{1/2}}{\pi}\Gamma\left(\frac{5}{3}\right)2^{2/3}\alpha n_0 a\frac{\rho}{\lambdabar}\gamma_0^{-2}(1-a^{-1/3})(B_0\Lambda)^{-1},$$
$$E_1=\frac{3^{5/2}\Gamma(5/3)}{2^{10/3}\pi}\alpha a n_0\gamma_0\approx 0{,}45\alpha a n_0\gamma_0. \tag{75}$$

As regards the density of the curvature photons, it is of the order of the density of the first-generation electrons and positrons, while the energy of the curvature radiation is $a/B_0\Lambda$ times less, in accordance with its steeper spectrum. However, when the first generation of particles is produced, second-generation photons are produced, their energy lying in the interval

$$k_{min}<k<k_ca^{-1}.$$

Their energy density is

$$E_2=\frac{9\Gamma(5/3)}{2^{1/3}\pi}\frac{\alpha n_0}{B_0^2\Lambda}\gamma_0 aQ\left(\frac{ak_{min}}{k_c}\right), \tag{76}$$

where

$$Q(x)=3.93\ln(1/x)-19.35(1-x^{1/3})$$
$$+5.44(1-x)-2{,}19(1-x^{4/3}).$$

It is easy to show that for the majority of pulsars the photon mean free path is too large for production of a third generation: $l\sim La^2\sim R$. Therefore, the production of synchrophotons and particles in pulsars is usually limited to two generations. It is possible to find $k_{\min}$ by equating the energy of the second-generation photons to the total energy loss (62):

$$E_2=n_0\gamma_0[1-(1+3\eta_m)^{-1/3}].$$

It follows from this that for not too strong fields, $B_0<0.8\alpha^{1/2}=0.07$,

$$k_{min}=\frac{k_c}{a}\left[1-1{,}27B_0\left(\frac{\Lambda}{a\alpha}\right)^{1/2}[1-(1+3\eta_m)^{-1/3}]^{1/2}\right].$$

The second-generation photons produce electrons and positrons of a second generation too, and their density is

$$n_2^\pm=\frac{1}{\pi^2}3\cdot 2^{2/3}\Gamma\left(\frac{5}{3}\right)\frac{\alpha n_0}{\Lambda B_0^2}\frac{\rho}{\lambdabar\gamma_0^2}a^2Q_1\left(\frac{ak_{min}}{k_c}\right).$$

Here

$$Q_1(x)=(x^{-1}-1)[3.93(x^{-1}-1)-9{,}67(x^{-2/3}-1)$$
$$+5{,}44\ln(1/x)-8{,}76(1-x^{1/3})].$$

This makes it possible to determine an important parameter of the theory—the multiplicity of the secondary plasma (see Ref. 1):

$$\lambda=\frac{n}{n_c}=0{,}27\frac{i}{1+K_m}\frac{\rho}{\lambdabar\gamma_0{}^2}B_0\left(\frac{\Lambda a}{\alpha}\right)^{1/2}[1-(1+3\eta_m)^{-1/3}]^{3/2}, \tag{77}$$

where $n\sim n_2^{\pm}$ is the plasma concentration. It follows from (77) that for ordinary conditions in pulsars $\lambda\sim 10^3$.

Thus, the expressions (72)–(76) describe the concentration, energy, and distribution functions of the particles and photons in the plasma multiplied in the pulsar magnetosphere. We note that the results of the theory agree qualitatively with the numerical Monte Carlo calculations of plasma multiplication made by Daugherty and Harding.[5] In addition, the currently available data of observations on the γ fluxes from the two most active pulsars, Crab and Vela, in the energy range 10^2–10^4 MeV (Refs. 13 and 14) permit them to be compared with the results of the theory [the expressions (64) and (72)]. The estimates obtained from the theory are found to agree well with the measured values. Detailed comparison of the theory with the observational data requires an extended analysis, which is beyond the scope of the present paper.

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[1)]We do not consider here superstrong magnetic fields, $B\gtrsim B_c=4.43\cdot10^{13}$ G, in which the path of a photon may not be straight (see Ref. 6).

[2)]Transverse drift may play a part in the motion of individual formations, for example, drifting subpulses.[11]

[3)]Note that the expressions (53)–(55) actually hold in stronger magnetic fields when the condition $B_0\lesssim 0.1$ of Eq. (22) is not satisfied. It is merely necessary to replace Λ in the expression by $2B_0^{-1}/3$, i.e., set $\varkappa=1$, which alters the dependence of ψ_0 on the magnetic field from the dependence (55) ($B_{12}^{1/7}$ instead of $B_{12}^{-1/7}$).

[4)]The polar cap is the region near the magnetic axis of the neutron star from which open magnetic field lines emanate, i.e., lines that intersect the "light surface." It is only along such lines in the pulsar magnetosphere that longitudinal electric currents can flow.[1]

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