Nonlinear instability theory for a diffusive neutral layer

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A nonlinear theory of kinetic instability of collisionless plasma in a self-consistent magnetic field with a neutral layer is investigated. The case of a diffusive neutral layer is considered. A linear theory is developed for an arbitrary angle of propagation of growing perturbations, and quasilinear relaxation effects in the plasma distribution accompanying the instability development are discussed. A nonlinear mechanism leading to the suppression of instability is discussed in general terms. The results can be used to estimate the dissipation of the energy of the magnetic field in the model of a neutral layer discussed in this paper.

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1. INTRODUCTION

The mechanism responsible for the reconnection of lines of force and the annihilation of a magnetic field in plasma configurations containing neutral layers (or lines) have long attracted the attention of astrophysicists specializing in the physics of the sun and the magnetosphere of the earth. Plasma processes developing in neutral layers may provide an effective mechanism for the transformation of magnetic-field energy into thermal and directed energy of particles.

Most of the results in this area have been obtained for collision-dominated plasma in the MHD approximation (see[1] and the bibliography therein). In the present paper, we consider collisionless plasma that can be described by the Vlasov equation. We consider the case of a diffusive neutral layer with $\rho_{Be,i}/L \ll 1$, where $\rho_{Be,i}$ are the Larmor radii of the particles and L is the thickness of the plasma layer (the linear theory for the opposite case of a thin layer is developed in[2]). Plasma containing a neutral layer of arbitrary but large thickness is always unstable against the development of the tearing mode.[3] The tearing mode is, in a sense, a universal instability (and, in many cases, the only instability) exhibited by plasma configurations including a neutral layer. For a sufficiently thin layer, there is the possible additional excitation of current instabilities, when the equilibrium current exceeds a certain critical value.

It will be convenient to start with the equilibrium state[4] described in Sec. 2. The well-known linear theory of tearing-mode instability is given in[5-7] for the most important case of propagation along the magnetic field, and this is generalized in Sec. 3 to the case of propagation at an arbitrary angle. The results (which are different from the incorrect results reported in[8]) are used to develop a quasilinear theory.

The quasilinear theory of the evolution of the tearing-mode instability has previously been described only in[9, 10]. In Sec. 4, we essentially reconsider the results reported there. In Sec. 4.1] we give a general account of the physics of the instability, which provides an indication of the direction in which the quasilinear effect should be looked for. Section 4.2 is devoted to quasilinear effects in the "internal" region (where the magnetic field can be neglected). It is shown that quasilinear diffusion in the internal region cannot prevent development of the instability. In Sec. 4.3 we consider the role of quasilinear processes in the external region, which again cannot rapidly suppress instability.

Finally, in Sec. 5 we consider nonlinear effects stabilizing the instability, in general terms. The physical nature of this stabilization process is connected with the capture by the magnetic field of large-scale fluctuations which grow during instability development. It is shown in this way that, in a time of the order of γ_e^{-1}, the plasma enters the quasistationary state characterized by the presence of a magnetic-field component perpendicular to the neutral layer. The escape of plasma, together with the reconnected lines of force along the neutral layer may maintain the instability at a low level in the case of a system of finite length.

2. STATE OF EQUILIBRIUM

We shall take the equilibrium plasma configuration described by Harris[4] as the initial state. In this configuration, the ions and electrons are described by Maxwell distribution functions with a constant velocity shift along the y axis:

$$f_{0j}=(\alpha_j/\pi)^{3/2}n(x)\exp\{-\alpha_j(v_x^2+(v_y-u_j)^2+v_z^2)\},\quad \alpha_j=m_j/2T_j=1/v_{Tj}^2. \tag{2.1}$$

The problem can be conveniently analyzed in the moving coordinate frame in which the electric field is $E_0=0$. This is ensured when the following condition is satisfied in the system:

$$u_i/T_i=-u_e/T_e. \tag{2.2}$$

The self-consistent magnetic field is parallel (or antiparallel) to the z axis and is zero on the x = 0 plane (Fig. 1):

$$B_z=B_0\,\mathrm{th}\,(x/L). \tag{2.3}$$

The plasma density distribution along the x axis is

$$n(x)=n_0\,\mathrm{ch}^{-2}(x/L). \tag{2.4}$$

The quantities B_0 and n_0 are related by the condition that the pressures are equal

$$B_0^2/8\pi=n_0(T_e+T_i), \tag{2.5}$$

where L is the characteristic size of a change in density,

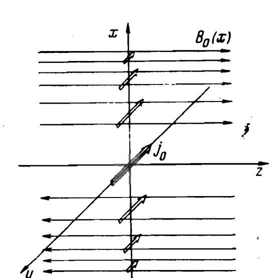

FIG. 1

and the magnetic field along the x axis and is given by

$$L=\left[\frac{4\pi e^2 n_0}{m_e c^2}\frac{u_e^2}{v_{Te}^2}\left(1+\frac{T_i}{T_e}\right)\right]^{-1/2}. \tag{2.6}$$

This configuration has all the features characteristic of plasma containing a neutral layer and is, at the same time, sufficiently simple and convenient for investigation. We therefore follow most other workers concerned with the tearing-mode instability of plasma containing a neutral layer[5-10] and choose the Harris configuration described above as the initial state of equilibrium.

3. LINEAR THEORY OF TEARING-MODE INSTABILITY

The linear growth rate of this instability was first estimated by Laval and Pellat.[5] Dobrowolny[7] obtained approximate dispersion equations and found the final solution of the problem for waves propagating along the magnetic field ($\mathbf{k} = k_z\mathbf{e}_z$). Although this case is the most important from the point of view of the linear theory, the development of a quasilinear theory requires knowledge of the wave spectrum at arbitrary angles. The solution of this problem is given below.

Linearizing the Vlasov equation, we obtain the linear addition to the distribution function in the form

$$f_{1j}=-\frac{e}{m_j c}\int_{-\infty}^{t}(\mathbf{E}_1 c+[\mathbf{v}'\times\mathbf{b}_1])\frac{\partial f_{0j}}{\partial\mathbf{v}'}dt'. \tag{3.1}$$

We must now introduce the potentials $\mathbf{A}$ and φ with the gauge div $\mathbf{A} = 0$. The perturbed quantities will be taken in the form of the Fourier harmonics in ω, y, z:

$$\Psi=\Psi(x)\exp\{-i\omega t+ik_y y+ik_z z\}. \tag{3.2}$$

The Maxwell equations for the potentials $\mathbf{A}$ and φ with the chosen gauge can be written in the form

$$\Delta\mathbf{A}=-(4\pi/c)\mathbf{j},\quad \Delta\varphi=-4\pi\rho, \tag{3.3}$$

where we have neglected displacement currents, and $\mathbf{j}$ and ρ are the perturbed current and charge densities.

It is shown in[11] that the integrals of the equations of motion of particles in the above problem have the property that the x component on the right-hand side of the first equation in (3.3) is zero, i.e., $j_x = 0$. This immediately yields $A_x = 0$ and the gauge equation assumes a simple and x-independent form, indicating the orthogonality of $\mathbf{k}$ and $\mathbf{A}$:

$$k_y A_y+k_z A_z=0. \tag{3.4}$$

In terms of the potentials, the equation for f_{1j} is

$$f_{1j}=\frac{2\alpha_j e_j f_{0j}}{m_j c}\left[A_y u_j-\varphi c+i(\omega-k_y u_j)\int_{-\infty}^{t}dt'(\mathbf{A}\mathbf{v}'-\varphi c)\right]. \tag{3.5}$$

It is shown in[7] that the potential φ can be neglected for low-frequency oscillations: $\varphi \sim \omega(\mathbf{A}\cdot\mathbf{v})/kv_T$. Moreover, it is also shown in[7] that very complicated particle orbits in the magnetic field $B_z = B_0\tanh(x/L)$ can be approximated by simplified orbits when the problem of tearing-mode instability is analyzed. Thus, in a thin layer for which $|x| < d_j \sim \sqrt{\rho_j L}$ (where $\rho_j = v_{Tj}m_j c/eB_0$), roughly speaking, the particles behave as if there were no magnetic field. The motion of the particles outside this internal region for $|x| > d_j$ takes the form of the usual Larmor rotation and a simultaneous magnetic drift of particles along the y axis due to the inhomogeneity of the magnetic field $B_z(x)$.

The quantities f_{1j} can be readily calculated in the internal region:

$$f_{1j}^{<}=\frac{2\alpha_j e_j f_{0j}}{m_j c}\left(A_y u_j-\frac{\omega-k_y u_j}{\omega-\mathbf{kv}}\mathbf{Av}\right). \tag{3.6}$$

If we determine the current component in the direction of $\mathbf{A}$, we obtain the equation

$$\frac{d^2A}{dx^2}-(k^2+V(x))A=0,\quad k=|\mathbf{k}|=(k_y^2+k_z^2)^{1/2}. \tag{3.7}$$

The potential in the internal region will be denoted by $V^{<}$. It can be calculated with the aid of (3.6), and turns out to consist of two parts, namely, the adiabatic part and the part connected with the denominator in (3.6).

The nonresonance adiabatic part is given by

$$\text{v. p.}\int e_i \mathbf{v} f_{1j}^{<}\,d^3v.$$

For an arbitrary angle θ' between $\mathbf{k}$ and the z axis, it is convenient to carry out the calculations in a rotated set of coordinates in which $\tilde{z} \parallel \mathbf{k}$. We then have

$$V_{NR}^{<}=-2L^{-2}\mathrm{ch}^{-2}(x/L)\cos 2\theta'. \tag{3.8}$$

The expression given by (3.8) includes a contribution due to both ions and electrons. It is important to note that the factor $\cos 2\theta'$ in the region with zero magnetic field is the same as that obtained in[12] for anisotropic instability in plasma with zero magnetic field. The resonance contribution to $V^{<}$ differs from the result obtained in[7] only by the convective addition to ω:

$$V_R^{<}=\sum_j\left(\frac{4\pi e^2 n_0}{m_j c^2}\mathrm{ch}^{-2}\frac{x}{L}(-i\sqrt{\pi})\frac{\omega-ku_j\sin\theta'}{kv_{Tj}}\right). \tag{3.9}$$

Let us now consider the external region. In (3.5), in which we have set $\varphi = 0$, the second term which is connected with the integral over the trajectory is now determined by the magnetic drift velocity $v_{Mj} \sim \rho_j v_{Tj}/L \sim u_j$. The resonance contribution to $V^{>}$ is governed only by this term and is small because $(u_j/v_{Tj})^2 = \epsilon_j^2 \ll 1$ and $V_R^{<}$ and $V_R^{>}/V_R^{<} \sim \epsilon_j^2$. The contribution of this term to $V_R^{\gtrless}$ is also negligible, so that we finally obtain

$$f_{1j}^{>}=\frac{2\alpha_j e_j f_{0j}}{m_j c}A_y u_j, \tag{3.10}$$

$$V_{NR}^{>}=-2L^{-2}\mathrm{ch}^{-2}(x/L)\cos^2\theta'. \tag{3.11}$$

Equation (3.7) is the one-dimensional Schroedinger equation with $E = -k^2$. We can use this fact to determine the relative role of contributions due to the terms $(V_R^{<})_{e,i}$, $(V_R^{\gtrless})_{e,i}$ in the development of the instability. It will be convenient, temporarily, to set $\theta' = 0$. On the instability boundary with $\omega = 0$, where $V_R^{\lesseqgtr} = 0$, the potential $V(x) = -2L^{-2}\cosh^{-2}(x/L)$ has the form of a well containing only one energy level, namely, $E = -L^{-2}$, and the corresponding symmetric solution has the form (see Fig. 2)

$$A(x)\sim P_1^{-1}\left(\mathrm{th}\frac{x}{L}\right)=\frac{1}{2}\mathrm{ch}^{-1}\frac{x}{L}, \tag{3.12}$$

where P_1^{-1} is the generalized Legendre function of order -1.

When $kL > 1$ ($E < -L^{-2}$), the plasma is stable. When the energy level in the well exists, and lies above $E_0 = -L^{-2}$ ($0 > E > E_0$), the plasma may exhibit instability. Schindler et al.[13] have shown that, in the general case, the Dobrowolny solution[7] is an expansion over a small deviation from the critical state on the instability boundary (in our case, the state with $k_0 L = 1$). Therefore, when we execute

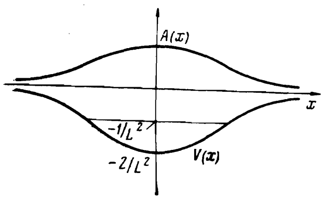

FIG. 2

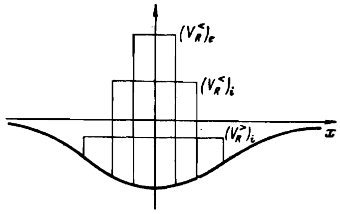

FIG. 3

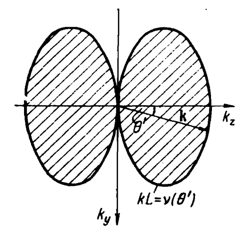

FIG. 4

the transition from states on the instability boundary to unstable states ($\omega \neq 0$, $V_R \neq 0$), the corrections to the energy E_0 associated with V_R should be small, and this determines the maximum possible values of ω.

Although it is readily shown that

$$(V_R^<)_i/(V_R^<)_e \sim (m/M)^{1/2}, \quad (V_R^>/V_R^<)_j \sim \varepsilon_j^2,$$

this does not necessarily mean (as concluded in[5-7]) that the dominant contribution is due to electrons in the internal region, i.e., $(V_R^<)_e$, since the sizes of the regions D in which these terms are significant are different (see Fig. 3):

$$\frac{[D(V_R^<)]_i}{[D(V_R^<)]_e} \sim \frac{d_i}{d_e} \sim \left(\frac{M}{m}\right)^{1/4}, \quad \left[\frac{D(V_R^>)}{D(V_R^<)}\right]_j \sim \frac{L}{d_j} \sim \frac{1}{\varepsilon_j^{1/2}}.$$

It is clear that the dimensionless parameter $VD^2(V)$ is the same for the internal terms. However, it can be shown that the perturbation introduced in the present case by a high barrier for a low-lying level depends on another dimensionless parameter, namely, $VD(V)/|E|^{1/2}$, and this parameter for electrons in the internal region is greater than for ions in this region by a factor of $(M/m)^{1/4}$, and greater by a factor of $(\epsilon_i)^{-3/2}(M/m)^{1/4}$ than for ions in the external region. It follows that the contribution of electrons in the internal region is, in fact, the most important.

It is readily seen that this conclusion remains valid for an arbitrary angle θ'. When $\theta' \neq 0$, the potential $V^>$ is given by (3.11) although it turns out that the terms $V_{NR}^>$ and $V_{NR}^<$ have a different dependence on θ'. Thus, $V_{NR}^< \ll V_R^<$ and $V_{NR}^<$ turns out to be unimportant in the internal region, so that the solution of the problem can be obtained by generalizing the results in[7].

In the internal region, the solution has the form

$$A^<(x) \sim \operatorname{ch}(V_R^< + k^2)^{1/2} x, \tag{3.13}$$

where $V^< \sim$ const.

In the external region, we take the solution which vanishes at $\pm\infty$:

$$A^>(x) = P_\nu^{-m}\left(\operatorname{th}\frac{x}{L}\right); \tag{3.14}$$

where $m = kL$, $\nu = 1/2[(1 + 8\cos^2\theta')^{1/2} - 1]$ [this is obtained from the condition $\nu(\nu + 1) = 2\cos^2\theta'$]; when $\theta' = 0$, we have $\nu = 1$ and the solution becomes identical with that obtained by Dobrowolny.[7]

The state at the limit of stability has the energy

$$E_* = -\nu^2 L^{-2} \quad (kL = \nu).$$

The energy level in the well for this state will, therefore, exist for any $\nu \neq 0$. This is in agreement with the principle established in,[13] which gives the sufficient condition for instability. Therefore, all waves propagating at an angle to the magnetic field other than the right-angle ($\theta' \neq \pm\pi/2$) are unstable (see diagram in Fig. 4). This result differs from that reported in[8] where only waves in a relatively narrow band of angles $\theta' < \epsilon_i^{1/4}$ are unstable.

We must now join the solutions in internal and external regions by equating the logarithmic derivatives of (3.13) and (3.14) at $x = d_e$. This yields the required dispersion relation for oscillations near the equilibrium state. The frequency and growth rate for the tearing-mode instability at an arbitrary angle of propagation are given by

$$\omega_R = ku_e \sin\theta', \tag{3.15}$$

$$\gamma = kv_{Te}\varepsilon_e^{3/2}\pi^{-1/2}(1 + T_i/T_e)\psi(m, \nu). \tag{3.16}$$

The angular dependence of the growth rate is described by the function:

$$\psi(m, \nu) = -(\nu + m)\Gamma\left(\frac{m+\nu}{2}\right)\Gamma\left(\frac{1+m-\nu}{2}\right)\Big/\Gamma\left(\frac{m-\nu}{2}\right)\Gamma\left(\frac{1+m+\nu}{2}\right); \tag{3.17}$$

where for $\nu = 1$ ($\theta' = 0$) the function $\psi(m, \nu)$ becomes identical with the expression $(1 - m^2)/m$, analogous to that obtained in[7]. Transition to this state at the limit of stability with $\gamma = 0$ corresponds to the denominator in (3.17) becoming infinite. This occurs for $m = \nu$, which, in turn, gives the well-known result $E_0 = -\nu^2 L^{-2}$. The instability is possible only for $m < \nu$. Figure 4 shows the polar diagram in the k plane. The region of stability is separated from the region of instability by the curve

$$(2kL + 1)^2 = 1 + 8\cos^2\theta'. \tag{3.18}$$

These results exhaust all the problems of the linear theory.

4. INFLUENCE OF QUASILINEAR EFFECTS

4.1. Physical Nature of Tearing-Mode Instability

To elucidate the role of quasilinear and nonlinear effects in the development of the tearing mode, we must first discuss the physical nature of this instability.

The tearing mode is a wave of negative energy. In plasma, the energy W_w of the wave consists of the energy of fluctuations in the magnetic field

$$W_M = \int \frac{b_1^2}{8\pi} d^3\mathbf{r}$$

and the energy of nonresonant particles, W_{NR}. It can be negative only in a nonequilibrium medium. The initial inhomogeneity of the plasma is connected with the shift (anisotropy) of the distribution function (2.1). It is well-known that the growth of negative-energy waves occurs during positive dissipation, i.e., when energy W_R is supplied to resonant particles.

Conservation of energy, in this case, can be written in the form

$$\Delta(W_{0M} + W_w + W_R) = 0, \tag{4.1}$$

where $W_{0M} = \int (B_0^2/8\pi) d^3\mathbf{r}$ is the energy of the initial magnetic-field configuration, $\Delta W_W = \Delta[W_M + (W_{kin})_{NR}] < 0$ is the energy of the wave in the medium, and $\Delta(W_{kin})_R > 0$ is the change in the kinetic energy of resonant particles.

In the linear approximation, when changes in f_0 can be neglected, we have W_{0M} = const and (4.1) becomes the energy principle first obtained in[5]. We then have

$$\Delta W_{kin}=(W_{kin})_R+(W_{kin})_{NR}=\int \mathbf{E}\mathbf{j}\,d^3r\,dt. \qquad (4.2)$$

In the linear approximation, the quantity given by (4.2) is given by

$$\sum_{\mathbf{k}}\int dx\frac{1}{8\pi}|A_{\mathbf{k}}|^2V(x),$$

where $V^{\gtrless}(x)$ are calculated in the preceding section.

During the development of the instability, quasilinear changes in f_{0j} may lead to a self-consistent change in the magnetic-field configuration, with the corresponding release or absorption of energy, and this leads to the violation of the simple energy principle.[5] Of course, to ensure that the change in the magnetic-field configuration occurs self-consistently with the change in f_{0j}, it is necessary that the problem be capable of being considered as a quasiequilibrium problem at each instant of time, and this, in turn, means that the characteristic time τ_{QL} of quasilinear relaxation must be much greater than τ_x, i.e., the characteristic time for the propagation of perturbations along the x axis.

According to the Le Chatelier principle, quasilinear effects must tend to remove the causes of instability, i.e., the anisotropy of the distribution function and the electron resonance contribution. We shall show below that the quasilinear effects do, in fact, act in this direction. However, they are relatively unimportant because they lead to the removal of anisotropy mainly in the internal region, where it is unimportant, and to the suppression of resonance effects in the external region, where their role is negligible anyway. In this respect, our results are essentially different from those in[9, 10].

Firstly, the quasilinear relaxation does not lead to the establishment of a plateau in the internal region. The instability in this region is essentially non-one-dimensional, as shown in Sec. 3, and the establishment of the plateau in the two-dimensional resonance region would require an infinite amount of energy, and is therefore impossible.[14] Secondly, quasilinear relaxation of nonresonant particles in the internal region can lead, at most, to the liquidation of anisotropy, i.e., $V^{<}_{NR} = -2L^{-2}$ is replaced by $V^{<}_{NR} \to 0$ at the beginning of the process. The formation of the potential $V^{<} \sim d_e^{-2} > 0$ by quasilinear effects, which lies at the basis of the results in[9], is impossible because this would be equivalent to the introduction of negative anisotropy into the distribution function, which would be greater by a factor of ϵ_e^{-1} than the original anisotropy.

4.2. Quasilinear Effect in the Internal Region

In the internal region $|x| < d_e$, where the effect of the magnetic field on particle orbits can be neglected, the quasilinear relaxation of tearing mode instability is, in principle, analogous to the well-known problem of the relaxation of anisotropic instability in plasma in the absence of a magnetic field[12, 15] (see also[9, 10, 14]). Differences between the two are connected only with the fact that the initial anisotropy in these problems has a different character, depending on the angular width of the region of unstable $\mathbf{k}$. According to (3.8), unstable angles of the vector $\mathbf{k}$ in plasma without a magnetic field are given by the condition $\cos 2\theta' > 0$ (whereas, for the tearing mode instability, all the angles θ' except for $\theta' = \pm\pi/2$ are unstable). In plasma without a magnetic field, the quasilinear equation has the well-known form[14]

$$\frac{\partial \bar f}{\partial t}+\left\langle\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{b}]\right)\frac{\partial f_i^{\bullet}}{\partial \mathbf{v}}\right\rangle=0, \qquad (4.3)$$

where the angle brackets represent averaging over all space.

Transforming to cylindric coordinates in $\mathbf{v}$-space (v_x, $v_\perp$, θ), we obtain

$$f_{1\mathbf{k}}=\frac{e}{mc}(A_\perp)_{\mathbf{k}}\frac{1}{\omega-k_\perp v_\perp\cos\vartheta}\hat Q_{\mathbf{k}}\bar f. \qquad (4.4)$$

In this expression, $\vartheta = \theta - \theta'$ and

$$\hat Q_{\mathbf{k}}=\left[\omega\sin\vartheta\frac{\partial}{\partial v_\perp}+\left(\frac{\omega\cos\vartheta}{v_\perp}-k_\perp\right)\frac{\partial}{\partial\theta}\right].$$

Hence, we have

$$\frac{\partial \bar f}{\partial t}=\frac{e^2}{m^2c^2}\sum_{\mathbf{k}}\hat Q_{\mathbf{k}}\frac{i|A_{\mathbf{k}}|^2}{\omega-k_\perp v_\perp\cos\vartheta}\hat Q_{\mathbf{k}}\bar f. \qquad (4.5)$$

If the spectrum of unstable $\mathbf{k}$ were one-dimensional, then

$$|A_{\mathbf{k}}|^2\sim|A_k|^2\delta(\theta-\theta'),$$

and diffusion in the (v_y, v_z) plane described by (4.5) would result in the formation of a new set of level lines in the distribution function. In the asymptotic state in which there is no diffusion, $\hat Q_{\mathbf{k}}\bar f_{QL} = 0$. Solving this partial differential equation, we can readily show that the required function f_{QL} should have the following level lines:

$$(v_\perp\cos\vartheta-\omega/k_\perp)^2+(v_\perp\sin\vartheta)^2=C. \qquad (4.6)$$

The quantity ω in this expression should be interpreted as $\omega_R = \mathrm{Re}\,\omega$. The distribution function f_{QL} has a plateau in the resonance region, i.e., all the level lines are parallel to $\mathbf{k}$ and, consequently, are orthogonal to the boundaries of the resonance region $|\omega_R - \mathbf{k}\cdot\mathbf{v}| = \gamma_{\mathbf{k}}$. The level lines of the function f_{QL} for a given angle θ' are shown in Fig. 5.

It is important to note that the shift of the resonance region relative to the origin may be quite large. When t = 0

$$\omega/k_\perp=\omega_R/k_\perp=u_e\sin\theta'.$$

When the anisotropy of the distribution function is reduced, the size of this shift is also reduced.

If we take into account the fact that, in reality, the wave spectrum in this problem is not one-dimensional but, in fact, very broad (all waves are unstable with the exception of those propagating at $\theta' = \pm\pi/2$), we find that, since the resonance regions corresponding to different

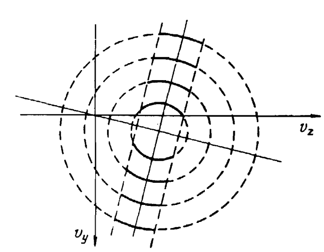

FIG. 5

θ' intersect, the establishment of the plateau which would be common for all angles would require an infinite amount of energy and this is, therefore, impossible.

We now recall that the angular spectrum of the waves is, roughly speaking, axially symmetric (with the exception of a narrow band of angles near $\theta' = \pm\pi/2$), so that we may conclude that quasilinear relaxation leads to the re-establishment of isotropy in the distribution function. However, the distribution function may assume a very complicated form during the relaxation process, and the assumption made in[12] that the distribution function retains a bi-Maxwellian character is not really satisfactory. Although the detailed character of this process will require further investigation, it is clear that the effective anisotropy should be reduced during the quasilinear relaxation process. This is confirmed by thermodynamic considerations[15] which show that an irreversible quasilinear process leads to the reestablishment of the Maxwell distribution function.

If nonlinear effects are ignored, then in the case of anisotropic instability, this relaxation leads to the exhaustion of the initial anisotropy and the transformation of the energy of resonant particles and the fluctuating magnetic field. For the tearing mode instability, most of the stored free energy feeding the instability is located in the external region and, therefore, the liquidation of anisotropy in the internal region leads not to the termination of instability but only to a small reduction in the total current in the direction of the y axis. When the rate at which the anisotropy is reduced in the internal region is much less than the rate of propagation of perturbations along the x axis (see Conclusions), this process can be looked upon as a quasiequilibrium process, and the reduction in the total current leads to a reduction in the magnetic field throughout the region.

In the region when there is no anisotropy, the equilibrium magnetic field must be zero. Whilst the instability is developing in the above fashion, the size of this internal region continues to increase because of the quasilinear removal of anisotropy at its edges, where the magnetic field is negligible and does not impede quasilinear relaxation. However, this particular quasilinear relaxation turns out to be suppressed by competing nonlinear mechanisms, described in Sec. 5.

4.3. Quasilinear Effects in the External Region

To complete the picture, let us consider the influence of quasilinear effects in the external region of tearing mode instability. When $|x| \gg d_e$, the magnetic field becomes sufficiently strong ($\omega/\omega_B \ll 1$, $\rho_B/L \ll 1$) and we can use the drift transport equation.[16] Transforming to the drift variables, and recalling that $\mu = v_\perp^2/2B(x)$ = const, we obtain

$$\frac{\partial f}{\partial t}+\left(\boldsymbol{\varepsilon}_0 v_\parallel-\frac{1}{\omega_B}\left[\boldsymbol{\varepsilon}_0\times\frac{\mathbf{F}}{m}\right]\right)\nabla f+\left(\boldsymbol{\varepsilon}_0\frac{\mathbf{F}}{m}\right)\frac{\partial f}{\partial v_\parallel}=0. \tag{4.7}$$

In the last equation, we have omitted the subscript j which identifies the particle species and have substituted

$$\boldsymbol{\varepsilon}_0=\mathbf{B}/|\mathbf{B}|,\quad \mathbf{F}=e\mathbf{E}-m\mu\nabla B,\quad \mu=\frac{v_\perp^2}{2B(x)}.$$

The contribution of the inertial force F_{in} to F is neglected. Linearizing (4.7), we obtain the correction f_{1k} to the distribution function in the form

$$f_{1\mathbf{k}}=-\frac{i}{\omega-k_z v_z-k_y v_M}\left[\left(\frac{cE_y}{B_0}+v_z\frac{b_x}{B_0}-\frac{\mu}{\omega_B}ik_y b_z\right)\frac{\partial \bar f}{\partial x}+\left(\frac{eE_z}{m}-\mu ik_z b_z-\mu b_x\varepsilon'\right)\frac{\partial f}{\partial v_\parallel}\right]. \tag{4.8}$$

In this expression

$$\varepsilon'=\frac{1}{B_0}\frac{dB_0}{dx},\quad v_M=\frac{\varepsilon' v_\perp^2}{2\omega_B}, \tag{4.9}$$

where v_M is the magnetic drift velocity of the particles. Terms containing b_z are the so-called nonquasiclassical terms which appear when the derivatives of the perturbed quantities with respect to x are taken into account.

The quasilinear equation is too complicated to be reproduced here in explicit form. We shall, however, summarize some of the conclusions regarding the diffusion described by this equation. The diffusion occurs only in the (x, v_z) space and although the condition that μ is conserved enables us to obtain the change in the total transverse energy of the particles, $v_\perp^2 = v_x^2 + v_y^2$, there is no relaxation of anisotropy in the (v_x, v_y) plane (this conclusion is valid to the same accuracy as the drift approximation used in this case).

It is well-known that this two-dimensional diffusion leads to the formation of a 'quasiplateau' in (x, v_z) space[17] and thus to the suppression of the instability.

The quasilinear operator in (4.8) can be simplified by noting that the nonquasiclassical terms are relatively unimportant in the external region. For $k_y < k_z$, we have

$$\hat{Q}_\mathbf{k}\sim\left[(\omega-k_z v_z)\frac{\partial}{\partial x}+\left(\mu k_z\frac{dB_0}{dx}\right)\frac{\partial}{\partial x}\right]. \tag{4.10}$$

Solving the equation $\hat{Q}_k f_{QL} = 0$, we find that the function f_{QL} should have level lines defined by:

$$v_\perp^2+(v_z-\omega/k_z)^2=C.$$

The relaxation of the initial function f_0, which has an anisotropy in the (v_x, v_y) plane, to the function f_{QL}, which is symmetric in this plane, is impossible because terms describing quasilinear diffusion in the (v_x, v_y) plane are negligible. Moreover, the resonance region $\omega - k_z v_z - k_y v_M = 0$ is found to be a paraboloid of revolution with the apex at the point $v_z = \omega/k_z$ when small but finite k_y are taken into account. Resonance regions intersect for different **k**, and the formation of the plateau is impossible.

Therefore, quasilinear effects in the external region do not lead to the suppression of the instability. The role of the quasilinear effect reduces to the "smearing out" of the inhomogeneous distribution of plasma along the x axis, and to a small reduction in the external contribution which, in any case, plays no appreciable role in instability development (Sec. 3).

5. INFLUENCE OF NONLINEAR EFFECTS ON THE DEVELOPMENT OF TEARING MODE INSTABILITY

We have shown above that the relaxation of tearing mode instability through quasilinear diffusion would lead to the smearing out of the initially localized sitribution of plasma over entire space. However, in the internal region, where the equilibrium magnetic field is small, the ratio $|b_1(x)/B_0(x)|$ can be as large as desired, even for small fluctuations $|b_1(0)/B_0(\infty)| \ll 1$. It is therefore clear that, in the internal region, an important role should be played by the nonlinear effect involving the perturbation of particle trajectories by fluctuating magnetic fields.

The exact solution of the problem of perturbation of electron trajectories in plasma without a magnetic field (internal region, where magnetic fields can be neglected) by the spectrum of random large-scale fluctuations in the magnetic field has not yet been obtained. It can be shown qualitatively that this effect should lead to the suppression of instability.

As noted in Sec. 4.1, the instability development is connected with the transfer of finite energy W_R to resonant particles in the internal region. Scattering of particles by growing fluctuations in the magnetic field in the case of small amplitudes, and the nonlinear capture of particles by fluctuations, which occurs at larger amplitudes (see below), will ensure that energy transfer between waves and particles is impossible, and this is one of the reasons for the nonlinear stabilization of developed instability.

In the chosen set of coordinates, growing fluctuations are polarized along the x axis. Let $\mathbf{k}^*$ be the characteristic wave number of fluctuations corresponding to complex frequency $\omega_{\mathbf{k}}^*$. We shall assume that the amplitude b of fluctuations has increased to a level such that the corresponding gyrofrequency and gyroradius of electrons, $\tilde{\omega}_{be} = eb/mc$ and $\tilde{\rho}_e = v_{Te}\tilde{\omega}_{be}^{-1}$, satisfy the conditions

$$\tilde{\omega}_{be} > |\omega_{\mathbf{k}^*}|, \quad \tilde{\rho}_e < |\mathbf{k}^*|^{-1}. \qquad (5.1)$$

It follows from (3.15) and (3.16) that the second inequality in (5.1) is the stronger

When (5.1) is satisfied, the motion of electrons in the wave can be looked upon as occurring in a quasistationary quasiuniform field. The particles can then be regarded as captured by the wave (Fig. 6), and their free motion in the internal region, which gives the resonance contribution (3.9), is replaced by Larmor rotation in the 'crests' of the magnetic field. The condition for the transition to this state is $\tilde{\rho}_e L^{-1} \sim 1$, since $|\mathbf{k}^*| \sim L^{-1}$. Although the electron contribution vanishes when (5.1) is satisfied, the actual contribution of electrons to the dispersion relation (3.7) can be shown to be

$$\tilde{V}_e^< \sim \varepsilon_e^{-2} L^{-2} (k_\perp \tilde{\rho}_e)^2. \qquad (5.2)$$

At the limit of validity of (5.2), where $k_\perp \tilde{\rho}_e \sim 1$, we have $\tilde{V}_e^< \sim \epsilon_e^{-2} L^{-2}$.

According to the calculations reported in[7], a positive hump appears at the center of the potential well with a width $\epsilon_e^{1/2} L$ and a height $\tilde{V}_*^< = \epsilon_e^{-1} L^{-2}$. This leads to the exclusion of the energy level from the well, and thus to the suppression of the instability. Since the well is shallow, the only possible solution is symmetric, so that there are no levels between the wall of the well and the central bump. The estimate given by (5.2) shows that after the transition to the state with captured electrons, the height of the positive hump at the center of the well turns out to be greater by a factor of ϵ_e^{-1} than that necessary for complete stabilization. When the positive-ion contribution $(V_R^<)_i$ is taken into account, this can only result in a still greater increase in the potential $V^<$. This substantial excess of the height of the hump in the potential $V^<$ over and above the critical height necessary for stabilization is connected with the fact that the instability should halt in magnetic fields smaller than those predicted by the condition $\tilde{\rho}_e L^{-1} < 1$. However, in this region, the approximation used above, in which the Larmor radius $\tilde{\rho}_e$ is assumed to be small, is invalid, and the exact solution of the problem must be used. It may therefore be supposed that the estimated critical level of fluctuations obtained from the condition $\tilde{\rho}_e L^{-1} \sim 1$, namely,

$$|b(0)/B_0(\infty)|_e^* \sim \varepsilon_e \qquad (5.3)$$

yields only the upper bound. At any rate, when (5.3) is satisfied, the instability is definitely stabilized.

The steady-state fluctuation level $|b/B_0|$ is small according to (5.3) and may exceed the quantity ϵ_e^3 obtained in[9]. However, in any case, if the steady-state energy fluctuations are small, this means that there is only a slight quasilinear relaxation of the initial state. In other words, nonlinear effects halt the instability so rapidly that quasilinear effects do not succeed in modifying substantially the initial state of the plasma during this time.

It follows from (5.2) that, as the amplitude of the fluctuations is increased further, the height of the potential hump (5.2) will decrease, and when $k_\perp \tilde{\rho}_e < \epsilon_e^{1/2}$ it becomes less than the critical value $V_*^< = \epsilon_e^{-1} L^{-2}$. The energy level can then again appear in the well, and the instability can grow. The resonant particles which remove energy from the waves can, in this case, be both ions (provided they are not captured by the waves) and electrons. The capture condition for ions can be formulated by analogy with (5.3) and is subject to the same restrictions:

$$|b/B_0|_i^* \sim \varepsilon_i. \qquad (5.4)$$

The condition that the ion instability will or will not occur depends, respectively, on the inequalities $\epsilon_i \gtrless \epsilon_e^{1/2}$. For a layer with a small current $\epsilon_i < (mT_e/MT_i)^{1/2}$, ion instability is impossible, and, in the steady state, the amplitude of the fluctuations in the transverse magnetic field is $b \lesssim \epsilon_e B_0$.

The nonlinear transition to ion instability is possible, in the case of a layer with a larger current, at least in principle. The state in which fluctuations transverse to the layer have the amplitude $b \lesssim \epsilon_e B_0$, and which is reached by the plasma as a result of the development of electron instability in a time of the order of γ_e^{-1}, is nonlinearly unstable against perturbations $b \sim \epsilon_e^{1/2} B_0$. Ion instability can then develop in the plasma, and the plasma will enter a state with transverse magnetic-field fluctuations $b \leq \epsilon_i B_0$ in a time of the order of γ_i^{-1} [γ_i can be found from (3.16) by replacing subscript e with subscript i].

The effectiveness of this mechanism in the case of anisotropic instability will require more detailed analysis because the steady-state energy of the fluctuations of which the upper bound can be estimated from the relation $|\mathbf{k}^*| v_{Te} \lesssim \tilde{\omega}_{be}$, where $|\mathbf{k}^*| \sim \omega_{pe}(\Delta T/T)^{1/2}/c$, turns out to be of the same order of magnitude as the energy of the initial anisotropy in this type of nonlinear stabilization, so that it is not clear a priori whether the nonlinear mechanism discussed above plays a significant role in the development of the instability.

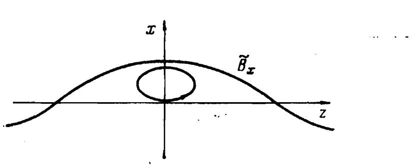

FIG. 6

6. CONCLUSIONS

The above analysis of the development of tearing-mode instability in collisionless plasma in a self-consistent magnetic field with a neutral layer has shown that, if the magnetic field component perpendicular to the layer is zero, this configuration is unstable against the electron tearing mode, and the development of this mode leads to the spontaneous crossing of the magnetic lines of force through the neutral layer. This configuration with reconnecting lines of force has a characteristic normal magnetic field component $b \lesssim \epsilon_e B_0$. When the current is sufficiently large, the configuration obtained as a result of the development of electron instability may turn out to be nonlinearly unstable against the development of the ion mode. The necessary condition for the development of the ion instability is $\epsilon_i > (mT_e/MT_i)^{1/2}$. Further rapid reconnection of the lines of force due to the development of the ion tearing mode comes to a halt only when the amplitude of the perturbations reaches $b \lesssim \epsilon_i B_0$.

The characteristic times for these processes, namely, $\gamma_i^{-1} \sim L v_{Ti}^{-1} \epsilon_i^{3/2}$ and $\gamma_e^{-1} \sim (M/m)^{-1/4} \gamma_i^{-1}$, are small in comparison with the characteristic time for the propagation of perturbations across the magnetic field $\tau_x \sim L v_A^{-1} \sim L v_{Ti}^{-1}$. Therefore, as noted in Sec. 4.1, the development of the instability can be looked upon as a quasiequilibrium process. In other words, we may suppose that, during the entire process, the configuration of the magnetic field varies self-consistently with the plasma distribution function. On the other hand, it turns out that the quasilinear relaxation of the distribution function in this process cannot play an appreciable role because nonlinear effects rapidly halt the development of the instability. This ensures that the dissipation of the magnetic-field energy in this model of a diffusive neutral layer turns out to be small and, in contrast to the case of a thin layer[2], does not lead to a substantial heating or acceleration of resonant particles in the internal region: $\Delta W_R/W_R \lesssim \epsilon_{e,i}^2$.

It is important to note that the estimates given by (5.3) can be used not only when the magnetic field perpendicular to the layer appears directly during the development of the instability, but also when the equilibrium magnetic field has a normal component B_n to the layer. The stabilization condition $B_n \lesssim \epsilon B_0$ obtained in this way differs from the condition for the suppression of instability obtained by Schindler,[18] i.e., $\tilde{\omega}_{Bn} > \gamma$ which, in fact, coincides with the first and weaker of the two conditions in (5.1). Moreover, the ion mode, the growth of which was used in,[18] may be absent altogether even for small normal components $B_n \lesssim \epsilon B_0$ because of the stabilizing effect of the contribution of the Larmor rotation of captured electrons. As shown above, the ion tearing mode can be nonlinearly excited only for sufficiently large currents j_{0y}.

Measurements of the normal component B_n in the tail of the magnetosphere yield values of the same order as are predicted by the estimate $B_n \lesssim \epsilon_i B_0$. This indicates that the ion instability may, in fact, occur in the earth's magnetosphere.

The development of tearing-mode instability in the above model leads to a fast spontaneous reconnection of lines of force through the neutral layer and, thereafter, the development of the instability is halted. The dissipation of the magnetic-field energy during the transition to the stable state is quite small. This enables us to conclude that any development of a model of the neutral layer, which would ensure effective annihilation of the magnetic-field energy and the acceleration of the particles, must take into account the forced reconnection of the lines of force. Since the development of the tearing-mode instability does not in itself produce this "forced reconnection," the electric field E_y in the direction of the current must be taken into account. The introduction of this current will ensure stationary drift of the plasma with velocity cE_y/B toward the neutral layer, and the subsequent flow of the plasma along the layer, thus leading to a solution with an effective stationary reconnection. This field can be external, i.e., applied to the plasma from outside, or associated with plasma inhomogeneity in the direction of the y axis[19, 20] or with the anomalous resistance of the plasma to the current j_{0y}.

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