field due to these ions.* Let E_r be the transverse field in the column when there are no negative ions and E_r^0 the transverse field in the case in which only surface recombination is assumed for the negative ions. Using the Schottky boundary condition and computing this ratio we find

$$E_r^0 / E_r = (1 + 2\varkappa b_n / b_e)^{-1}. \quad (9)$$

A measurement of the transverse fields makes it possible to estimate the concentration of charged particles and thus to obtain information on the relative importance of surface recombination and volume recombination.

The destruction of negative ions in collisions should also play a decisive role in negative-ion column kinetics. We propose to examine this question in the future.

*A. A. Zaitsev has indicated a possible method of estimating the transverse field. Since the curvature of the stratum is determined by the magnitude of the transverse gradient, by studying its behavior when electro-negative gases are added it is possible to estimate the relative change in the transverse gradient.

In conclusion we wish to express our gratitude to Professor V. L. Granovskii and Docent A. A. Zaitsev with whom we had a number of illuminating discussions and to E. V. Korotkov for help in carrying out the work.

[1] R. Holm, Z. Physik **75**, 171 (1932).
[2] R. Seeliger and R. Kruschke, Physik Z. **34**, 883 (1933).
[3] A. von Engel and M. Steenbeck, Elektrische gasentladungen, ihre physik und technik, Vol. I, (Berlin, 1934).
[4] R. Seeliger, Ann. Physik **6**, 93 (1949).
[5] V. L. Granovskii, Электрический разряд в газах (Electrical Discharges in Gases) Vol. I (1952).
[6] G. Ecker, Proc. Phys. Soc. (London) **B67**, 485 (1954).

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181

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MONTE CARLO CALCULATION OF AN ELECTRON-PHOTON CASCADE IN LEAD

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A statistical-probability method, based on random trials, is given for using a model to calculate the effects in an electron-photon cascade initiated in a layer of lead by a γ-ray quantum. This is a modification of the Monte Carlo method. Some typical curves calculated in this way are presented. Two methods are indicated for carrying out this sort of calculations by the use of high-speed electronic calculating machines.

WE have used a method of random trials, which is a modification of the well known Monte Carlo method,[1-3] for the calculation of the electron-photon cascade shower produced in a lead plate of thickness 0.5 cm by a γ-ray quantum of energy 500 Mev. The problem reduces to a stepwise reproduction of the picture of the natural process of the production of a cascade shower by a single primary particle. The "feeding in" of a large number of primary particles gives the distributions in energy and angle of the electrons, positrons, and γ-ray quanta emerging from the plate.

We have followed essentially the method of "drawing" given in the papers of Chavchanidze.[4,5] In the drawing we used tables of random numbers,[6] which make it unnecessary to use machines to ob-

tain such data.

Omitting the details,* we shall outline the method for calculating the cascade.

The distance traversed in the lead by the γ-ray quantum before its first interaction is determined by drawing. The drawing is based on the curve of the relation of total cross-section to quantum energy.

Then a drawing is made to determine the "fate" of the quantum. In the case of pair production, the energy of the positron is found by drawing, and this also fixes the energy of the electron. Drawings are made for the distances they traverse before interaction, as in the case of the γ-ray quantum. Then the amount of ionization loss is determined, and thus the energies of the particles at their subsequent collisions. A separate drawing gives the correction for multiple scattering. If the electron and positron have not traveled outside the plate their further fate is traced out.

To find by drawing the energies of Bremsstrahlung quanta, we used a method based on the construction of unnormalized integral curves with nonuniform scales for the arguments.

The drawing of the scattering angles was carried out without including correlation between the scattering angles of quantum and electron.

In the case of annihilation the angle of emission of one of the photons in the center-of-mass system is found by drawing, and this fixes that of the second photon. (The formulas for the angles and energies can be found in the book by Heitler.[7])

The results of the calculation are presented in the form of energy distribution (Fig. 1) and angle distribution (Fig. 2) curves for electrons, positrons, and photons, and also the so-called probability for "zero electrons" as a function of the aperture angle of the cone of observation (Fig. 3). The probability of "zero electrons" is defined as the probability that the developing cascade initiated by a single γ-ray quantum does not give any particle $e^{\pm}$ inside a prescribed cone with aperture angle ϑ. Here it is assumed that an electron or positron having energy less than 10 Mev either does not emerge from the lead plate of thickness 0.5 cm or else is not registered by the apparatus.

The data obtained also make it possible to find these distributions for the intermediate thicknesses 0.1, 0.2, 0.3, and 0.4 cm.

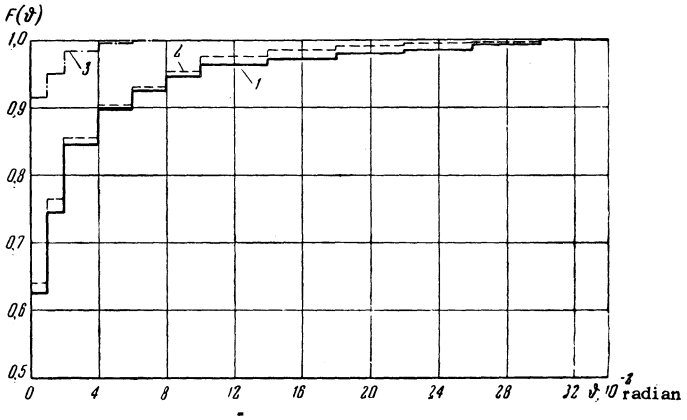

FIG. 2. Integrated angle-of-emergence distributions of electrons 1, positrons 2, and photons 3

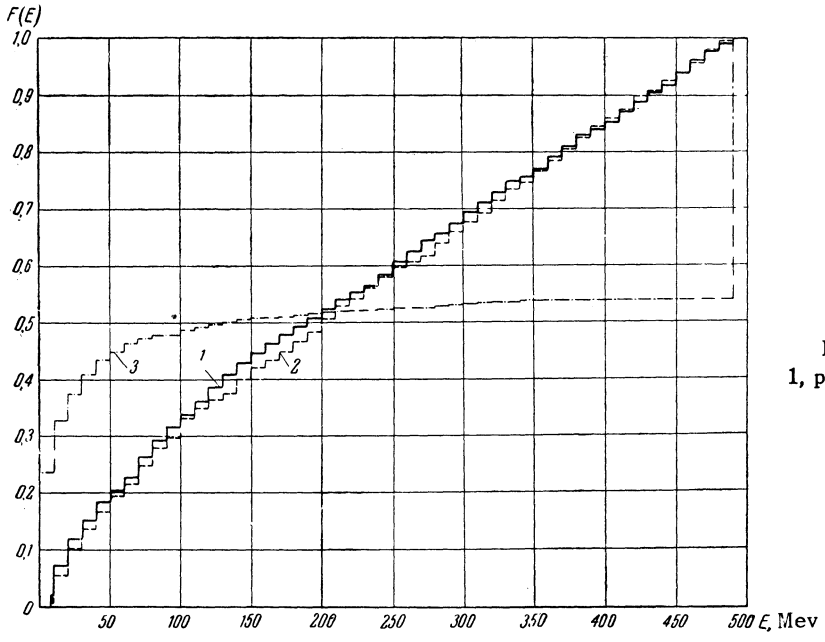

FIG. 1. Integrated energy distributions of electrons 1, positrons 2, and photons 3.

*This research will be published in full in Volume 5 of Trudy Instituta Fiziki Akademii Nauk Gruzinskoi S. S. R.

The amount of work for the calculation of the electron-photon cascade is extraordinarily large.

The largest amount of time is taken by the passage from one set of data obtained by drawing to the input data for the next drawing. The practical carrying out of such calculations requires the use of discrete-action high-speed electronic computing machines. For this purpose it has been found that it is quite unnecessary to remodel the existing machines. All that is required is appropriate programming. We shall state two methods of programming.

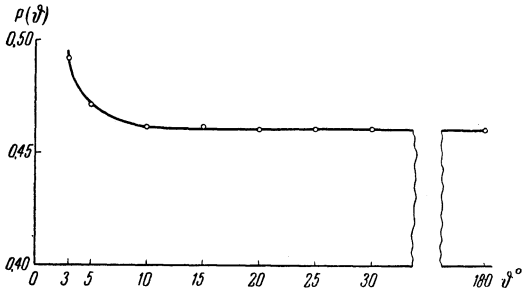

FIG. 3. Probability $P(\vartheta)$ that a cascade initiated by a single γ-ray quantum does not give any electron or positron inside a prescribed cone with aperture angle ϑ.

The distinguishing feature of the first method is that the arguments of the previously prescribed probability functions are stored in the permanent memory in a special order. Let us suppose that we have a diagram of the curve $P = F(x)$. This function can be represented by means of a set of pairs of numbers of the form $x_i, P_i = F(x_i)$, where i ranges from 1 to N. Sufficient accuracy for practical purposes can be obtained with $N \sim 300 - 500$. The statement of one member of a pair uniquely determines the other member. Therefore we need store in a definite section of the permanent memory device (MD) of our electronic computer only the arguments of the function. Suppose that for the storage of the numbers $x_i \epsilon M(x)$ (the set of arguments) we have set aside the counters with addresses beginning with a and ending with a + 1000. In the present case the entering of the numbers in the MD is carried out after a preliminary setting up of a certain sequence of values from among the x_i. A random number from the table of random numbers[6] is set equal to $F(x_b)$ and thus we have fixed a value of x_b, the other member of the pair x_b, $F(x_b)$. This number is entered in the MD under the address a + 1. The next random number supplies us with a value x_c, which is entered under the address a + 2, and so on. In this way 1000 numbers are put into the MD. The command for the use of the next of the random values $x_i \epsilon M(x)$ is taken care of by the use of these addresses in order, i.e., first a + 1, then a + 2, and so on. After the addresses are used up, the command can call for a displacement of the first address.

The second method of programming allows us to enter the function $F(x)$ in the MD in the usual way. But now the random numbers are stored in a special way in the counters under the various addresses of the MD: each number x_i is stored in the counter of the MD with the address $F(x_i)$. This type of storage is analogous to the storage of previously specified functions. The programming calls for choosing an address by taking a random number out of the table of random numbers. If the addresses take values from α to $\alpha + 1000$, then the program will call for the addresses $\alpha + p_i$, where the p_i are three-figure random numbers obtained by the programmer by reading the table of random numbers in a certain way. Thus a line of the program with a three-address code will have the following form:

$$\text{CRC} \qquad \alpha + p_i \qquad \beta \qquad \gamma$$

Here CRC means "command for a random choice". The CRC takes the number from the address $\alpha + p_i$ (α being a previously fixed number), adds it to the number at address β (this can also be zero) and sends the result to address γ.

It appears to us that the effectiveness of calculations by the method of random trials is fully manifested only when use is made of electronic computing machines, with a special procedure for programming and for entering the data in the memory device.

We take this occasion to express our gratitude to A. V. Tagviashvili, B. I. Bondarev, L. L. Esakiia, G. A. Gogadze, M. E. Pepel' man, and G. A. Almanov, who have taken part in the actual performance of the calculation.

The work presented here was carried out on the suggestion of Professor V. P. Dzhelepov and his group, in connection with the need for an estimate of the probability of failure of electrons and positrons to emerge from a thin layer of lead. The writers thank Professor Dzhelepov and his collaborators for their friendly attitude and their interest in the work.

[1] S. Ulam and J. von Neumann, Bull. Am. Math. Soc. **53**, 1120 (1947).

[2] R. R. Wilson, Phys. Rev. **86**, 261 (1952).

[3] Bernardini, Booth, and Lindenbaum, Phys. Rev. **88**, 1017 (1952).

[4] V. V. Chavchanidze, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 629 (1955).

[5] V. V. Chavchanidze, Trudy Inst. Fiz. Akad.

Nauk Gruz. SSR **3**, 105 (1955).
[6] M. Kadyrov, Таблицы случайных чисел (Tables of Random Numbers) Central Asian State University Press, Tashkent 1936.
[7] W. Heitler, The Quantum Theory of Radiation (Oxford, 1953).

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182

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ON THE RELATIONS BETWEEN THE CROSS SECTIONS FOR MULTIPLE PRODUCTION OF PIONS

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Isotopic relations have been obtained between the cross sections of transformation of a pion and a γ-quantum into three pions during collisions between the pion or γ-quantum and nucleons or deuterons. It is shown that near the threshold of the reaction, and also for production of pions with identical momenta, some auxiliary relations hold.

1. Relations have been obtained previously by the author between the cross sections of transformation of a single pion into two during collisions of pions with nuclei and deuterons.[1] Close to the reaction threshold, and also in the production of two pions with identical momenta, some auxiliary relations hold. In the present note it is shown that some auxiliary relations also hold between the cross sections of the various reactions of production of three pions during collisions of pions with nucleons and deuterons close to the threshold, or in the formation of pions with identical momenta.

Let us consider the collision of pions with nucleons (n) as the result of which the pion is transformed into three pions, i.e., the reaction

$$\pi + n \to n' + \pi' + \pi'' + \pi'''. \qquad (1)$$

The initial state is the superposition of states with isotopic spin T equal to $\frac{1}{2}$ and $\frac{3}{2}$. The wave function of the system of three pions is a superposition of states with isotopic spin t, equal to 0, 1, 2, 3. The wave function of the final state with total isotopic spin $T = \frac{1}{2}$ and $\frac{3}{2}$ can be constructed from the wave function of the nucleon (isotopic spin $\frac{1}{2}$) and from the wave function of the system of three pions with isotopic spin $t = 0$, 1, 2. The state with isotopic spin $t = 3$ is forbidden.

We denote by A_t^T, B_t^T, f_t^T, g_t^T and h_t^T the amplitudes of transitions into states with total isotopic spin T and isotopic spin of the system of three pions t and with a definite (see below) type of symmetry of the coordinate wave function of the pions.

Since the pions obey Bose statistics, then the type of symmetry of the coordinate part of the wave function and isotopic spin for the system of pions is determined by one and the same Young scheme.[2] The symmetry of the amplitude A_t^T is determined by a Young scheme consisting entirely of one row. The symmetric state of the system of three pions relative to an arbitrary pair of permutations of the charge variables is a state with total isotopic spin t equal to unity. Therefore, we have in all two different amplitudes, $A_1^{1/2}$ and $A_1^{3/2}$.

The symmetry of the amplitude B_t^T is determined by a Young scheme consisting of a single column. Since the antisymmetric state of the system of three pions relative to an arbitrary pair of permutations of the charge variables is the state with $t = 0$, then we have just the single amplitude $B_0^{1/2}$

The type of symmetry of the amplitudes f_t^T, g_t^T and h_t^T is determined by a Young scheme consisting of two rows. In his case t can take on the values 1 and 2.